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Homoclinic Cycle and Homoclinic Bifurcations of a Predator-prey Model with Impulsive State Feedback Control

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Abstract In this paper, the homoclinic bifurcation of a predator-prey system with impulsive state feedback control is investigated. By using the geometry theory of semi-continuous dynamic systems, the existences of order-1 homoclinic cycle and order-1 periodic solution are obtained. Then the stability of order-1 periodic solution is studied. At last, an example is presented to illustrate the main results.

Keywords Semi-continuous dynamic system, Successor function, Order-1 homoclinic cycle, Homoclinic bifurcation, Order-1 periodic solution.

MSC(2010) 34C23, 34C37, 34D15.

1. Introduction and model formulation

Since the mid 1920s, Vito Volterra and Alfred James Lotka proposed a groundbreaking model of the interaction between predators and prey [8, 17], researchers have conducted extensive research on predation, reciprocity, and competition mechanisms in recent years. A common research method is to study the evolutionary relationship between predators and prey by establishing suitable mathematical models. Then many mathematical models consisting of differential equations have been established and studied [3, 9, 15, 19, 20, 24, 26]. Some of them are represented by impulsive differential equations [11, 12, 21, 23]. Impulsive differential equations are a basic model for studying the process of a sudden change in the state of a system variable [1, 6, 27]. This sudden change is called a pulse. Systems with pulses that depend on the value of a variable in the systems are called the state-dependent impulsive system, which has become an important topic of impulsive differential equations and has been widely concerned by researchers [4, 5, 7, 13, 14, 16, 18, 22, 28–30].

Cui and Chen [2] proposed a mathematical model with functional response and undercrowding effect as follows,

$$\begin{cases} \dfrac{dx}{dt} = \dfrac{a}{k}x(x-L)(k-x) - \dfrac{bxy}{1+hx}, \\ \dfrac{dy}{dt} = -cy + \dfrac{dxy}{1+hx}, \end{cases} \tag{1.1}$$

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where x, y represent the population density of prey and predator, respectively. a, L, k, b, h, c, d are positive constants and $L < k$.

Yuan [25] considered the crowding effect in predator and a simple functional response based on system (1.1) as follows,

$$\begin{cases} \dfrac{dx}{dt} = ax(x-L)(k-x) - bxy, \\ \dfrac{dy}{dt} = -cy + dxy - \alpha y^2, \end{cases} \tag{1.2}$$

where x, y represent the population density of prey and predator. b represents the predation rate, c is the natural mortality rate of predators, d is the rate at which predator takes prey and then converts it to its own growth. α is death rate due to crowding effect. We nondimensionalize the system (1.2) with the following scaling,

$$t = \frac{\tau}{ak^2}, x = ku, y = \frac{ak^2}{b}v, p = \frac{L}{k}, m = \frac{kd}{c}, n = \frac{a\alpha k^2}{bc}, r = \frac{c}{ak^2}.$$

For the sake of convenience, we still use t to denote the change of time, then the system (1.2) will be transferred to

$$\begin{cases} \dfrac{du}{dt} = u(u-p)(1-u) - uv, \\ \dfrac{dv}{dt} = -rv(1-mu+nv), \end{cases} \tag{1.3}$$

In this paper, we consider the pulse state feedback control system based on model (1.3) as follows,

$$\begin{cases} \left.\begin{aligned} \dfrac{du}{dt} &= u(u-p)(1-u) - uv, \\ \dfrac{dv}{dt} &= -rv(1-mu+nv), \end{aligned}\right\} v \neq h, \\ \left.\begin{aligned} \Delta u &= -q_1 u, \\ \Delta v &= -q_2 v, \end{aligned}\right\} v = h. \end{cases} \tag{1.4}$$

It is obvious that $0 < p < 1$. $\Delta u = u(t^+) - u(t), \Delta v = v(t^+) - v(t)$. Considering the biological meaning, we will consider the solution of system (1.3) in region $R_+^2 = \{(x, y)|x \geq 0, y \geq 0\}$.

The organization of this paper is as follows. Some definitions and lemmas are presented in the section 2. We qualitatively analyze the system (1.3) in section 3. In section 4, we consider the existence and stability of order one periodic solution of system (1.4). In section 5, numerical simulations are carried out to illustrate the analytical results. We give a brief conclusion in section 6.

2. Preliminaries

In this section, we will introduce some notations, definitions and lemmas of the geometric theory of semi-continuous dynamic system, which will be useful for the following discussions. The following definitions and lemmas of semi-continuous dynamic system come from Chen et al. [1], Wei and Chen [22] and Pang and Chen [10].

Definition 2.1. Consider the following state dependent impulsive differential system

$$\begin{cases} \dfrac{dx}{dt} = P(x,y), \quad \dfrac{dy}{dt} = Q(x,y), \quad (x,y) \notin M\{x,y\}, \\ \Delta x = \alpha(x,y), \quad \Delta y = \beta(x,y), \quad (x,y) \in M\{x,y\}. \end{cases} \tag{2.1}$$

The solution mapping of system (2.1) is called as the semi-continuous dynamical system denoted by (Ω, f, φ, M), where $(x,y) \in \Omega \subset R_+^2, f = f(w;t)$ is the semi-continuous dynamical system mapping with initial point $w = (x_0, y_0) \notin M$, the sets M and N are called the impulse set and phase set, which are lines or curves on R_+^2. The continuous function $\varphi : M \to N$ is called impulse mapping.

Remark 2.1. System (2.1) constitutes a semi-continuous dynamic system (Ω, f, φ, M), where $\Omega = R_+^2 = \{(u,v)|u \geq 0, v \geq 0\}, M = \{(u,v) \in R_+^2|u \geq 0, v = h_2\}, \varphi : (u,v) \in M \to ((1-q_1)u, (1-q_2)h_2) \in R_+^2, N = \varphi(M) = \{(u,v) \in R_+^2|u \geq 0, v = (1-q_2)h_2\}$.

Definition 2.2. If there exists a point $P \in N$ and $T > 0$ such that $f(P,T) = Q \in M$ and $\varphi(Q) = \varphi(f(P,T)) = P \in N$, then $f(P,t)$ is called order-1 periodic solution.

Definition 2.3. The trajectory $f(P,t)$ combining with impulse line QP is called order-1 cycle. If the order-1 cycle has a singularity, then the order-1 cycle is called order-1 singular cycle. Furthermore, if the order-1 cycle only has a saddle, then the order-1 singular cycle is called order-1 homoclinic cycle.

Definition 2.4. We assume that G is a bounded closed simple connected region, which has the following properties:

(i) Impulse set M is a simple connected bounded closed straight line segments or curve segments which don't contain closed branch;

(ii) The boundaries AD, BC and AB of region G are non-tangent arcs of semi-continuous dynamical system (2.1). The boundary CD is the impulse set of system (2.1), its phase set satisfies $\varphi(CD) \subseteq AB$;

(iii) The orientation of the vector fields of semi-continuous dynamical system (2.1) on the AD,BC and AB point to the internal of region G. There are no equilibriums on the boundaries and also in the internal of region G of semi-continuous dynamical system (2.1).

Then region G is called Bendixson's region of semi-continuous dynamical system (2.1).

Lemma 2.1. *(Bendixson theorem of semi-continuous dynamical system) If region G is Bendixson's region of semi-continuous dynamical system (2.1), then there exists at least an order-1 periodic solution in the internal of region G (see Figure 1).*

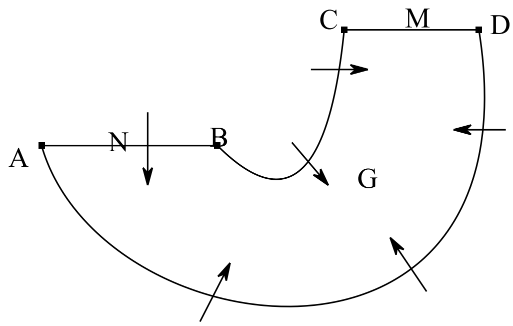

Figure 1. Bendixion region of semi-continuous dynamical system. Figure 1 is reproduced from C. Wei and L. Chen [22], [under the Creative Commons Attribution License/public domain].

Next, we will give the definition of successor function of system (2.1).

Definition 2.5. Suppose $g : N \to N$ be a map. Let $P \in N$ be the initial mapping point, for any $P \in N$, there exists a $t_1 > 0$ such that $F(P) = f(P, t_1) = P_1 \in M, P_1^+ = \varphi(P_1) \in N$. Then, function $g(P) = l(P_1^+) - l(P)$ is the successor function of point P, and the point P_1^+ is called the successor point of P, where $l(P)$ and $l(P_1^+)$ are the abscissas of point P and P_1^+, respectively. (see Figure 2).

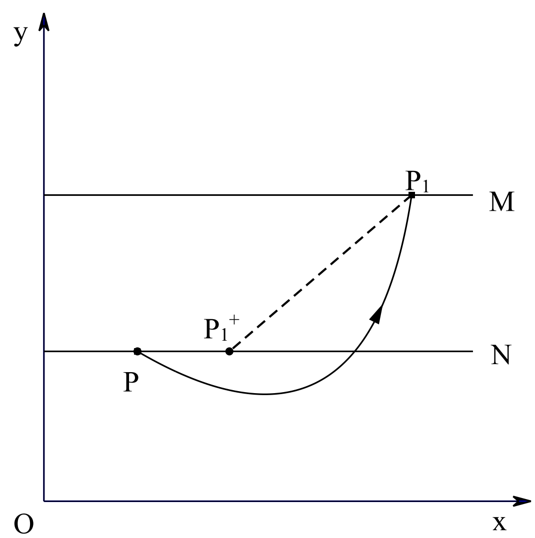

Figure 2. Successor function. Figure 2 is reproduced from C. Wei and L. Chen [22], [under the Creative Commons Attribution License/public domain].

Definition 2.6. Suppose $\Gamma = f(P, t)$ is an order-1 periodic solution of system (2.1). If for any $\varepsilon > 0$, there must exist $\delta > 0$ and $t_0 \geq 0$, such that for any point $P_1 \in \cup(P, \delta) \bigcap N$, we have $\rho(f(P_1, t), \Gamma) < \varepsilon$ for $t > t_0$, then we call the order-1 periodic solution Γ is orbitally asymptotically stable.

3. Some basic results about existence and stability of the equilibria in model (1.3)

In [25], the author has given whole results about existence and stability of the equilibria in model (1.3), here we summary them in the following theorem in this section.

Theorem 3.1. *(i) System* (1.3) *always has a trivial equilibrium* $O(0,0)$ *and two semi-trivial equilibria* $P(p,0), Q(1,0)$.

(ii) System (1.3) *has two positive equilibria* $E_0 = (u_0, v_0)$ *and* $E_1 = (u_1, v_1)$ *if and only if* $\frac{1}{m} < p < 1$, *where* $u_i, v_i (i = 0, 1)$ *satisfy*

$$nu_i^2 + [m - n(1+p)]u_i + np - 1 = 0, v_i = (u_i - p)(1 - u_i). \tag{3.1}$$

(iii) If $\Delta = 0$, *then system* (1.3) *has a unique positive equilibrium* $E_2(u_2, v_2)$ *(i.e.* E_1 *coincides with* E_0*), where* $u_2 = \frac{-(m-n(1+p))+\sqrt{\Delta}}{2n}, v_2 = \frac{mu_2-1}{n}$ *and* $\Delta = [m - n(1+p)]^2 - 4n(np - 1)$.

(iv) If $\frac{1}{m} = p$, *then system* (1.3) *has only one positive equilibrium point* E_0 *(i.e.* E_1 *coincides with* P*), where* $u_0 = 1 - \frac{1}{np}, v_0 = \frac{1}{np}(1 - p - \frac{1}{np})$.

We always suppose $nr - 2 < 0, 0 < p < 1, r > 0$ hold in the whole paper.
Denote

$$M = \{(m,n,r,p)|nr < 2, np^2 < np-1 < n, m < (1-p)n, \frac{1}{p} < m < n(1+p)-2\sqrt{n(np-1)}\}$$

and

$$M_1 = \{(m,n,r,p) \in M, 1 < m < m^*\},$$

where

$$m^* = \frac{(1+p)[(nr-2)(nr-1) - 2np] + (nr - 2 + 2np)\sqrt{(1-p)^2(nr-1)^2 + 4p}}{2nrp(nr-2)},$$

$$T = (1+p)u_0 - 2u_0^2 - nrv_0.$$

Theorem 3.2. $O(0,0)$ *is a stable node,* $P(p,0)$ *is a unstable node and both* $Q(1,0)$ *and* $E_1(u_1, v_1)$ *are saddles. If* $(H) : (m,n,r,p) \in M_1$ *and* $T < 0$ *hold, then* $E_0(u_0, v_0)$ *is a stable focus or node.*

Theorem 3.3. *The solution of system* (1.3) *is ultimately bounded.*

Proof. Given the initial conditions

$$u(t_0) = u_0 > 0, v(t_0) = v_0 > 0. \tag{3.2}$$

Let $(u(t), v(t))$ be a solution of system (1.3) satisfied initial conditions (3.2). We will build a area Ω with a boundary such that $(u(t), v(t)) \in \Omega$ for initial point $(u(t_0), v(t_0))$ and $t > T$, where $T > 0$ is large (see Figure 3). Since point $Q(1,0)$ is a saddle point, for line $l_1 : u - 1 = 0$ passing through Q, we have

$$\left.\frac{dl_1}{dt}\right|_{l_1=0} = \left.\frac{du}{dt}\right|_{u=1} = -v < 0.$$

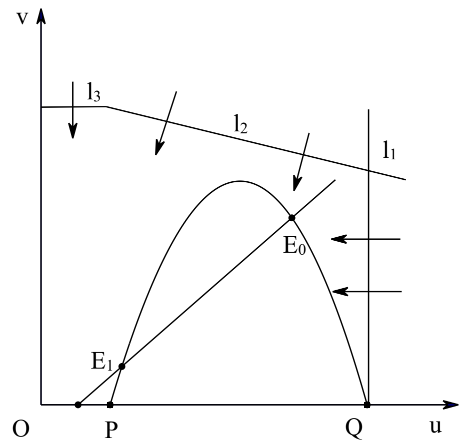

Figure 3. The solution of system (1.3) is bounded.

Thus the line l_1 is a segment without contact and orbit of system (1.3) goes across it from the right. On the other hand, define in the first quadrant: $l_2 : mru+v-M=0$, where $M > \frac{m(1-p)^2}{4} + mr$, then we have

$$\begin{aligned}\frac{dl_2}{dt}|_{l_2=0} =& mru[u(u-p)(1-u)-uv]-rv(1-mu+nv)\\ =& r[mu(u-p)(1-u)-nv^2-v]\\ <& r[m(u-p)(1-u)-v]\\ =& r[m(u-p)(1-u)+mru-M]\\ \leq& r[m(u-p)(1-u)+mr-M]\\ <& r[\frac{m(1-p)^2}{4}-M],\end{aligned}$$

where $u < 1$.

Denote $l_3 : v - \frac{M-r}{rn} = 0$, easily we get $u < \frac{M}{mr} - \frac{M-r}{mnr^2}$, then we have $u < \frac{M}{mr}$. Therefore we get

$$\frac{dl_3}{dt}|_{l_3=0} = -rv(1-mu+nv) = -rv\frac{M-mur}{r} < 0.$$

Then there exists a area Ω with a boundary being composed of $u = 0, v = 0, l_1, l_2$ and l_3 such that $(u(t), v(t)) \in \Omega$ for initial point $(u(t_0), v(t_0))$ and $t > T$, where $T > 0$ is large. This completes the proof. □

4. Dynamics analysis of the model with impulsive state feedback control

In this section, we will investigate the existence of order-1 period solution by using the method of successor functions, as well as the orbitally asymptotical stability of periodic solutions by using the monotonicity of the successor function. Obliviously, $L_1 : 1 - mu + nv = 0$ and $v = 0$ are Y-nullclines, $L_2 : v = (u-p)(1-u)$ and $u = 0$ are two X-nullclines.

4.1. Homoclinic cycle and Homoclinic bifurcations of model (1.4) about parameter q_1

In this section, we will discuss the existence of order-1 homoclinic cycle of system (1.3), and we choose q_1 as the control parameter.

Theorem 4.1. *When the condition (H) holds, then there exists $q_1^* \in (0,1)$ such that system* (1.4) *has an order-1 homoclinic cycle.*

Proof. In model (1.4), since E_1 is a saddle point, then there must be an unstable manifold Γ_A and a stable manifold Γ_B, where Γ_A leaves the saddle point E_1 and Γ_A enters the saddle point E_1. According to the property of the trajectory of the system (1.3) and Theorem 3.3, we get that the unstable manifold is bound to intersect with the impulse set M, and the intersection point is denoted as $A(u_A, v_A)$. We denote the intersection of isocline $L_1 : \frac{dv}{dt} = 0$ and the pulse set M as C, and its coordinate is $C(u_C, v_C)$. The intersection of phase set N and stable manifold Γ_B is denote as $B(u_B, v_B)$, the intersection of N and L_1 is $D(u_D, v_D)$ (see Figure 4). Since the property of the trajectory of system (1.3), the unstable manifold Γ_A is above L_2, and the stable manifold Γ_B is below $L_1 : \frac{dv}{dt} = 0$. Since the pulse function $\psi(u, q_1) = (1 - q_1)u$ monotonically increases about u and monotonously decreases about q, so there must exist $q_1^* \in (0,1)$ such that $\psi(u_A, p^*) = (1 - q_1^*)u_A = u_B$, and AB, BE_1, E_1A formed a homoclinic cycle. The proof is completed. □

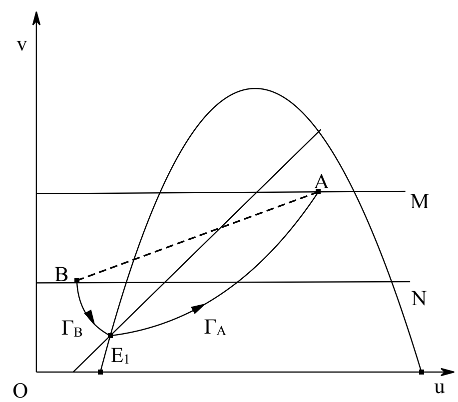

Figure 4. The existence of Order-1 homoclinic cycle.

Theorem 4.2. *When the conditions (H) and $\varphi(u_A, q_1) = u_{D_1} \leq u_D, \varphi(u_C, q_1) = u_{B_1} \geq u_B$ hold, then the homoclinic cycle of system* (1.4) *disappears and bifurcates an order-1 periodic solution and the solution is unique.*

Proof. Suppose the impulsive function transfers the point A into the point D, and transfers the point C into the point B_1, that is $\psi(u_A, q_1) = (1 - q_1)u_A = u_{D_1}$, $\psi(u_C, q_1) = (1-q_1)u_C = u_{B_1}$. If $q < q_1^*$, we have $u_{D_1} > u_{B_1}$. Since $u_B \leq \psi(u_C, q_1) = u_{B_1}$ and $u_D \geq \psi(u_A, q_1) = u_{D_1}$, we get $u_D \geq u_{D_1} \geq u_{B_1} \geq u_B$. Thus the Bendixson region G is formed by AC, CD, DB, BE_1, E_1A, where CD is a part of isoclinal L_1, $D_1B_1 \subset DB$, B_1E_1 is a part of Γ_B and E_1A is a part of Γ_A. By Lemma 2.1 and Theorem 3.3 ,we know that system (1.4) has an order-1 periodic solution (see Figure 5). Next we will discuss the uniqueness of the order-1 periodic solution. Select two points I, J in the phase set BD arbitrarily, where $u_{B_1} < u_J < u_I < u_{D_1}$. Let

$F(I) = I_1 \in M, F(J) = J_1 \in M$, then I_1, J_1 jump to $I_1^+, J_1^+ \in N$ after the pulse effect. Due to $u_J < u_I$, then $u_{I_1} < u_{J_1}$ and $u_{I_1^+} = (1-q_1)u_{I_1}, u_{J_1^+} = (1-q_1)u_{J_1}$, thus we have $u_{I_1^+} < u_{J_1^+}$. Obviously, I_1^+ is the successor point of J_1^+. Therefore, we have $g(I) - g(J) = (u_{I_1^*} - u_I) - (u_{J_1^*} - u_J) = (u_J - u_I) + (u_{I_1^*} - u_{J_1^*}) < 0$, that is, $g(q_1)$ is monotonically decreasing in B_1D_1. Hence, there must exist a point H such that $g(H) = 0$. That is, the order-1 periodic solution of system (1.4) is unique (see Figure 6). □

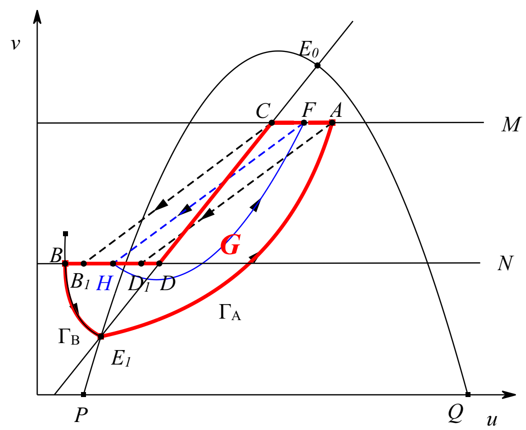

Figure 5. The existence of order-1 periodic solution of system (1.4).

4.2. Orbitally asymptotical stability of the order-1 periodic solution

Theorem 4.3. *When conditions* (H) *and* $\varphi(u_A, q_1) = u_{D_1} \le u_D, \varphi(u_C, q_1) = u_{B_1} \ge u_B$ *hold, then the order-1 periodic solution of system* (1.4) *is orbitally asymptotically stable.*

Proof. From Theorem 4.2, we know that the order-1 periodic solution of system (1.3) is unique and it passes through the point $H \in N$, where $u_{B_1} < u_H < u_{D_1}$. Let $F(D_1) = C_1 \in M$, then the point C_1 jumps to C_1^+ after the pulse effect. We have $u_{B_1} < u_{C_1^+} < u_H$ such that $F(C_1^+) = C_2 \in M$. Then we can get $u_H < u_{C_2^+} < u_{D_1}$ and $u_{C_1} < u_F < u_{C_2} < u_A$ because trajectories cannot intersect, where H is the pulse point of the order-1 periodic solution.

Similarly, let $F(C_2^+) = C_3 \in M$, we can easily get $u_{C_1^+} < u_{C_3^+} < u_H < u_{C_2^+}$ and $u_{C_1} < u_{C_3} < u_F < u_{C_2}$. Repeat the above steps, we get a sequence $\{C_k\}_{k=1,2,\dots}$ of pulse set M and a sequence $\{C_k^+\}_{k=1,2,\dots}$ of phase set N and $F(C_k^+) = C_{k+1}, u_{C_{2k-1}^+} < u_{C_{2k+1}^+} < u_H < u_{C_{2k}^+} < u_{C_{2k-2}^+}$. That is, we get $u_{B_1} < u_{C_1^+} < u_{C_3^+} < \ldots < u_{C_{2k-1}^+} < u_{C_{2k+1}^+} < u_H$ and $u_{D_1} > u_{C_2}^+ > u_{C_4^+} > \ldots > u_{C_{2k}^+} > u_{C_{2(k+1)}^+} > \ldots > u_H$. Therefore, sequence $\{C_k^+\}_{k=1,2,\dots}$ of phase set N is monotonically decreasing and the sequence $\{C_k\}_{k=1,2,\dots}$ of pulse set M is monotonically increasing. In the same time, we have $u_{C_{2k}^+} \to u_H$ when $k \to \infty$ and $u_{C_{2k-1}^+} \to u_H$ when $k \to \infty$. Arbitrarily choose a point $H_0 \in C_1^+D_1$ which is different from H, let $u_H < u_{H_0} < u_{D_1}$ (otherwise, $u_{C_1^+}+ < u_{H_0} < u_H$, the discussion is similar).

Then there must exist a integer k such that $u_{C^+_{2k+1}} < u_{H_0} < u_{C^+_{2k}}$. The trajectory starting from H_0 will experience the pulse effect indefinitely. We denote the phase point corresponding to the l^{th} pulse effect as $H_l, l = 0, 1, 2\ldots$, then for any l, we have $u_{C^+_{2(k+l)+1}} < u_{H_{2l+1}} < u_{C^+_{2(k+l+1)+1}}$, therefore $\{u_{H_2l}\}_{l=0,1,2,\ldots}$ is monotonically decreasing, and $\{u_{H_2l+1}\}_{l=0,1,2,\ldots}$ is monotonically increasing. Thus, after the pulse effect, the successor point of the phase point is attracted to H, which means that the order-1 periodic solution of the system (1.4) is orbitally asymptotically stable (see Figure 7). $\square$

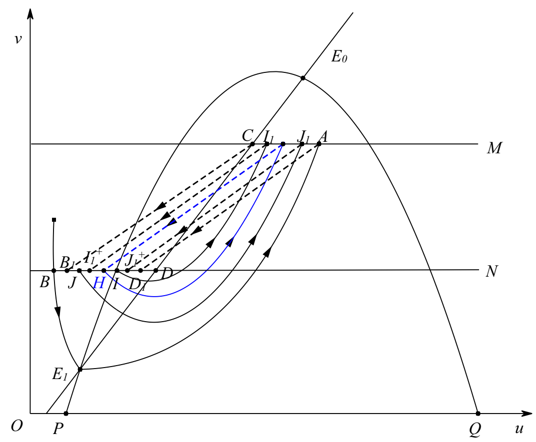

Figure 6. The monotonicity of the successor function g in the segment B_1D_1.

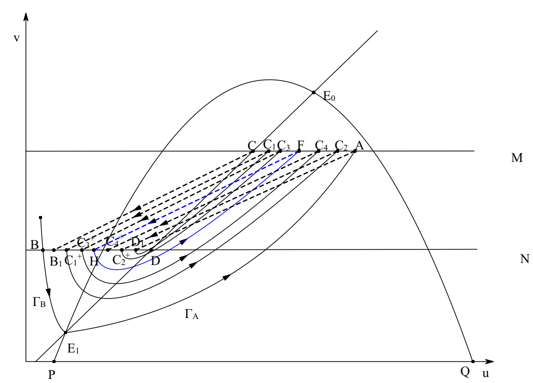

Figure 7. Illustration of orbitally asymptotical stability of the order-1 periodic solution of system (1.4)

5. Numerical simulations

In this section, we give an example with some numerical simulations to illustrate the theoretical results. First, we consider the system neglecting impulsive state feedback control, let $n = 15, r = 0.3, p = 0.4, m = 3.24$, simple calculations show that system (1.3) have five equilibria, i.e., $O(0, 0), P(p, 0), Q(1, 0), E_0 = (0.4611, 0.03293)$ and $E_1 = (0.7228, 0.08947)$ (see Figure 3) and among them, $O(0, 0)$ is a stable node,

$P(p, 0)$ is a unstable node and both $Q(1, 0)$, $E_1(u_1, v_1)$ are saddles and $E_0(u_0, v_0)$ is a stable focus or a node.

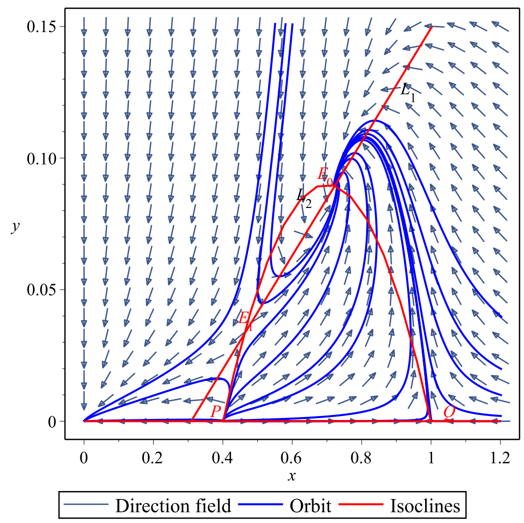

Figure 8. Phase diagram of system (1.3) with $n = 15, r = 0.3, p = 0.4, m = 3.24$.

Next we consider the impulsive state feedback control system and let $h = 0.08$, we get the system as follows,

$$\begin{cases} \left.\begin{aligned} \frac{du}{dt} &= u(u-0.4)(1-u) - uv, \\ \frac{dv}{dt} &= -0.3v(1-3.24u+15v), \end{aligned}\right\} v \neq 0.08, \\ \left.\begin{aligned} \Delta u &= -q_1 u, \\ \Delta v &= -q_2 v, \end{aligned}\right\} v = h. \end{cases} \tag{5.1}$$

First we take parameter as $q_1 = 0.311, q_2 = 0.25$, according to Remark 2.1, $\{v = 0.08\}$ is the impulse set M, $\{v = 0.06\}$ is the phase set N. Then system (5.1) has a homoclinic cycle composed of the unstable manifold Γ_A, the stable manifold Γ_B and the pulse straight line (see the red curve in Figure 9, where the initial value is $u_0 = 0.465, v_0 = 0.0329$).

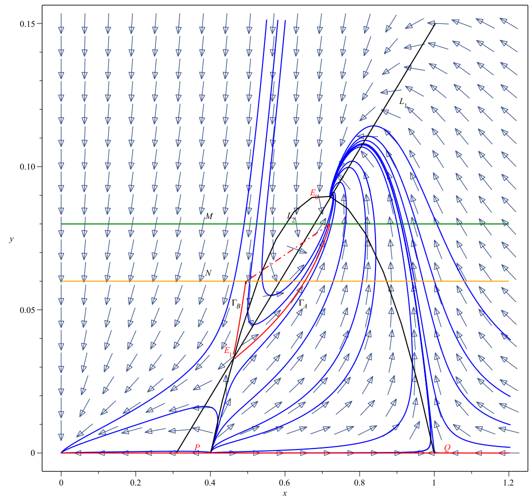

Figure 9. Order-1 homoclinic cycle of system (5.1) with $q_1 = 0.311, q_2 = 0.25$.

If we fix parameter $q_2 = 0.25$ and change parameter q_1 from 0.311 to 0.25, numerical simulation shows that the order-1 homoclinic cycle disappears and an order-1 periodic solution is bifurcated out form the order-1 homoclinic cycle, which is shown in Figure 10(see the purple curve), where the initial value is $u_0 = 0.465, v_0 = 0.0329$.

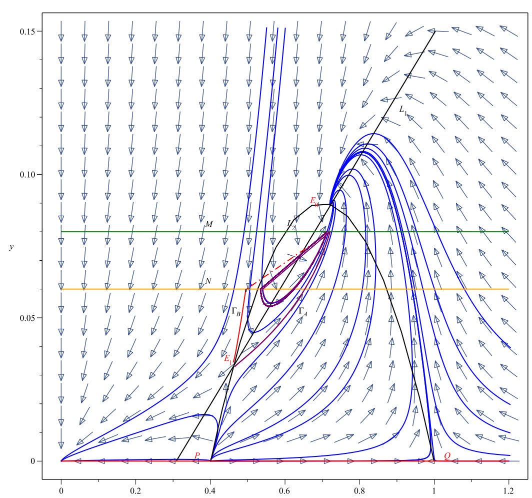

Figure 10. Order-1 homoclinic bifurcation of system (5.1) with $q_1 = 0.25, q_2 = 0.25, u_0 = 0.465, v_0 = 0.0329$.

If we change the initial value, for example, let $u_0 = 0.55, v_0 = 0.07$, we can also get an order-1 periodic solution of system (5.1) (see the red curve in in Figure 11).

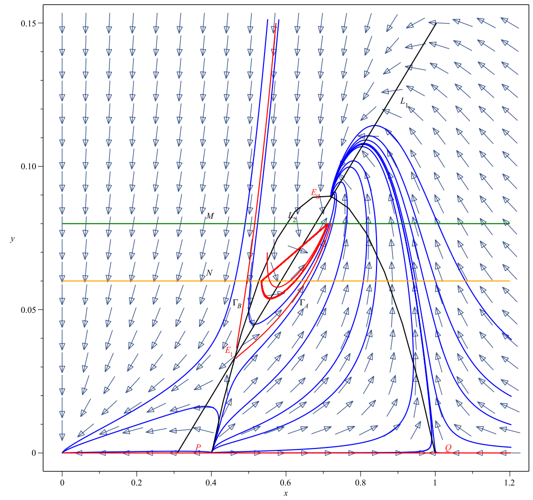

Figure 11. Order-1 periodic solution of system (5.1) with $q_1 = 0.25, q_2 = 0.25, u_0 = 0.55, v_0 = 0.07$.

6. Conclusion

In this paper, a predator-prey system with impulsive state feedback control is proposed and analyzed. The results show that the system under impulsive state feedback control can exhibit rich dynamics, for example, the system has a unique order-1 homoclinic cycle, moreover, by choosing q_1 as the control parameter, we prove that the order-1 homoclinic cycle disappears and bifurcates an orbitally asymptotical stable order-1 periodic solution when q_1 changes.

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References

[1] L. Chen, X. Liang and Y. Pei, *The periodic solutions of the impulsive state feedback dynamical system*, Communications in Mathematical Biology and Neuroscience, 2018, 2018. Article ID 14.

[2] J. Cui and L. Chen, *Mathematical research of the system with functional response and undercrowding effect*, Chinese Journal of Engineering Mathematics, 1995, 12(1), 27–36. In Chinese.

[3] Y. Du, P. Y. H. Pang and M. Wang, *Qualitative analysis of a prey–predator model with stage structure for the predator*, SIAM Journal on Applied Mathematics, 2008, 69(2), 596–620.

[4] H. Guo, L. Chen and X. Song, *Geometric properties of solution of a cylindrical dynamic system with impulsive state feedback control*, Nonlinear Analysis: Hybrid Systems, 2015, 15, 98–111.

[5] G. Jiang, Q. Lu and L. Qian, *Complex dynamics of a Holling type II prey-predator system with state feedback control*, Chaos, Solitons & Fractals, 2007, 31(2), 448–461.

[6] V. Lakshmikantham, D. D. Bainov and P. Simeonov, *Theory of Impulsive Differential Equations*, World Scientific, Singapore, 1989.

[7] Z. Li, L. Chen and Z. Liu, *Periodic solution of a chemostat model with variable yield and impulsive state feedback control*, Applied Mathematical Modelling, 2012, 36(3), 1255–1266.

[8] A. J. Lotka, *Elements of Physical Biology*, Williams & Wilkins, Baltimore, 1925.

[9] X. Meng, F. Li and S. Gao, *Global analysis and numerical simulations of a novel stochastic eco-epidemiological model with time delay*, Applied Mathematics and Computation, 2018, 339, 701–726.

[10] G. Pang and L. Chen, *Periodic solution of the system with impulsive state feedback control*, Nonlinear Dynamics, 2014, 78(1), 743–753.

[11] R. Shi, X. Jiang and L. Chen, *A predator–prey model with disease in the prey and two impulses for integrated pest management*, Applied Mathematical Modelling, 2009, 33(5), 2248–2256.

[12] X. Song and Y. Li, *Dynamic complexities of a Holling II two-prey one–predator system with impulsive effect*, Chaos, Solitons and Fractals, 2007, 33(2), 463–478.

[13] K. Sun, T. Zhang and Y. Tian, *Dynamics analysis and control optimization of a pest management predator-prey model with an integrated control strategy*, Applied Mathematics and Computation, 2017, 292, 253–271.

[14] S. Tang and R. A. Cheke, *State-dependent impulsive models of integrated pest management (ipm) strategies and their dynamic consequences*, Journal of Mathematical Biology, 2005, 50(3), 257–292.

[15] A. J. Terry, *Predator–prey models with component Allee effect for predator reproduction*, Journal of Mathematical Biology, 2015, 71(6), 1325–1352.

[16] Y. Tian, K. Sun and L. Chen, *Modelling and qualitative analysis of a predator-prey system with state-dependent impulsive effects*, Mathematics and Computers in Simulation, 2011, 82(2), 318–331.

[17] V. Volterra, *Leçons sur la théorie mathématique de la lutte pour la vie*, Gauthier-Villars, Paris, 1931.

[18] J. Wang, H. Cheng, Y. Li and X. Zhang, *The geometrical analysis of a predator-prey model with multi-state dependent impulsive*, Journal of Applied Analysis & Computation, 2018, 8(2), 427–442.

[19] J. Wang, J. Shi and J. Wei, *Predator–prey system with strong Allee effect in prey*, Journal of Mathematical Biology, 2011, 62(3), 291–331.

[20] X. Wang, L. Zanette and X. Zou, *Modelling the fear effect in predator–prey interactions*, Journal of Mathematical Biology, 2016, 73(5), 1179–1204.

[21] C. Wei and L. Chen, *Homoclinic bifurcation of prey-predator model with impulsive state feedback control*, Applied Mathematics and Computation, 2014, 237, 282–292.

[22] C. Wei and L. Chen, *Periodic solution and heteroclinic bifurcation in a predator-prey system with Allee effect and impulsive harvesting*, Nonlinear Dynamics, 2014, 76(2), 1109–1117.

[23] R. Wu, X. Zou and K. Wang, *Asymptotic behavior of a stochastic non-autonomous predator–prey model with impulsive perturbations*, Communications in Nonlinear Science and Numerical Simulation, 2015, 20(3), 965–974.

[24] D. Xiao and L. S. Jennings, *Bifurcations of a ratio-dependent predator-prey system with constant rate harvesting*, SIAM Journal on Applied Mathematics, 2005, 65(3), 737–753.

[25] M. Yuan, *Qualitative analysis of volterra predator-prey system under two equilibrium points*, Shihezi University of Agriculture, 1996, 14(4), 73–80. In Chinese.

[26] S. Zhang, X. Meng, T. Feng and T. Zhang, *Dynamics analysis and numerical simulations of a stochastic non-autonomous predator–prey system with impulsive effects*, Nonlinear Analysis: Hybrid Systems, 2017, 26, 19–37.

[27] T. Zhang, N. Gao, T. Wang et al., *Global dynamics of a model for treating microorganisms in sewage by periodically adding microbial flocculants*, Mathematical Biosciences and Engineering, 2020, 17(1), 179–201.

[28] T. Zhang, W. Ma, X. Meng and T. Zhang, *Periodic solution of a prey-predator model with nonlinear state feedback control*, Applied Mathematics and Computation, 2015, 266, 95–107.

[29] T. Zhang, T. Xu, J. Wang et al., *Geometrical analysis of a pest management model in food-limited environments with nonlinear impulsive state feedback control*, Journal of Applied Analysis & Computation, 2019, 9(6), 1–17.

[30] L. Zhao, L. Chen and Q. Zhang, *The geometrical analysis of a predator-prey model with two state impulses*, Mathematical Biosciences, 2012, 238(2), 55–64.