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Fast Image Reconstruction Research Based on H∞ Filtering for Electrical Resistance Tomography ⋆

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Abstract

In order to improve the image reconstructed quality affected by soft filed feature and the speed of dynamic on-line data processing in Electrical Resistance Tomography, we propose a fast image reconstruction algorithm based on H∞ filtering theory. Mainly, on the H∞ filtering principle, a dynamic system is formulated firstly, whose inputs have unknown disturbances including noise errors and model errors, and the outputs have the estimation errors. Then, making the H∞ norm of this dynamic system as a cost function, a fast H∞ filtering algorithm is proposed whose criterion is to guarantee that the worst-cast effect of disturbance on estimation error is smaller than a given boundary. Experimental work was carried out for three typical flow distributions. Results showed that H∞ filter method improves the resolution of the reconstructed images and gains the strong robustness and anti-interference performance in unknown interference noise conditions. In addition, it dramatically reduces the computational time compared with the traditional Gauss-Newton iterative and Kalman filter methods. Therefore, the method is suitable for on-line multiphase flow measurement.

Keywords: Electrical Resistance Tomography; H∞ Filtering; Image Reconstruction

1 Introduction

In recent years, certain attempts have been made to adapt computerized tomography techniques to the needs of multiphase flow measurement. Electrical Resistance Tomography (ERT) have great potential for two-phase flow measurement, biomedical engineering, geophysical prospecting, etc [1-4]. The image reconstruction processing is inverse problem which will directly affect the medium distribution results. As a non-linear problem, ERT generally adopts some strategies to get a result. The existing image reconstruction algorithms have Truncated Singular Value Decomposition (TSVD), TV regularization algorithm and Newton-Ralph algorithm, etc [5-9],

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their purposes are to calculate the minimization residual errors through least squares estimate. Reconstruction algorithms can also be seen as a state estimation. The Kalman filter is an effective state estimation method. The attractiveness of the Kalman filter lies in the fact that it is the one estimator that results in the smallest possible standard deviation of the estimation error. That is, the Kalman filter is the minimum variance estimator if the noise is Gaussian [10]. But when the external disturbance signal is strong in ERT measurement system, the system model about external noise disturbance have certain uncertainty. Generally, the particle filter algorithm has a very good robustness. But numerous sampling particles are used to accurate approximation of the posterior distribution, the computational complexity is usually higher. Therefore, to solve the problems of high computational complexity and long computational time, we apply a high speed image reconstruction algorithm to gain optimal estimation for image reconstruction system. The method is based on optimal H∞ control theory whose criterion is to guarantee that the worst-cast effect of disturbance on estimation error is smaller than a given boundary.

2 CT Model for ERT

ERT system is composed of hardware parts which include the sensor array, analog switches, data acquisition system, and the computer to generate images. In ERT, the current patterns are injected into the unknown object through electrodes and the corresponding voltages on the surface of the object are measured. This procedure is repeated for all the electrode pairs. Then, each dataset is interpreted by image reconstruction algorithms to compute a cross-sectional image reconstruction. The composition of ERT system is shown in Fig. 1.

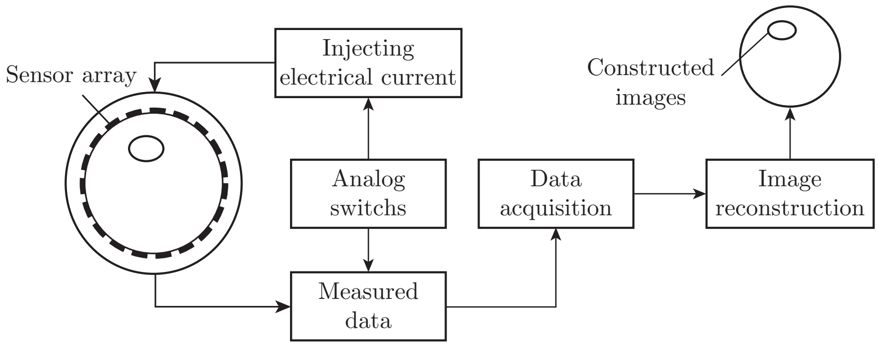

Fig. 1: The composition of ERT system

The electrical conductivity σ and electrical potential distribution Φ are governed by the Maxwell relations in ERT, leading to the equation:

$$\bigtriangledown(\sigma \bigtriangledown \Phi) = 0. \, \text{in}\, \Omega \tag{1}$$

where Ω represents the sensing field, boundary conditions for Eq. (1) are calculated as follows:

$$\sigma \frac{\partial \Phi}{\partial n} = \begin{cases} I \\ -I \end{cases}, \ \text{in}\, \partial\Omega_+, \partial\Omega_- \tag{2}$$

$$\partial + z \cdot \sigma \frac{\partial \Phi}{\partial n} = V. \quad \text{in}\, \partial\Omega \tag{3}$$

In these equations, where $\partial\Phi_+$ and $\partial\Omega_-$ are the surfaces of the source and sink electrodes, n is the vector normal the sensor periphery $\partial\Omega$, z is the contact impedance, I is the injected current and V are the potentials on the electrodes surface $\partial\Omega_V$. ERT problems are composed of the forward problem and the inverse problem. The forward problem is to determine voltage measurements. The image reconstruction process is the solution of the ERT inverse problem. Through the analysis of the sensitive field, nonlinear physical model can be linearized and normalized, the approximation linear equation is as follows:

$$V = J \cdot G + e \tag{4}$$

where, V is $n \times 1$ measurement vector matrix, J is $n \times m$ sensitivity coefficient matrix, G is $m \times 1$ matrix which relates to gray vector, and e is measurement error (n and m represent independent measuring electrode couple number and the pixel number).

3 Method

3.1 The Discrete Time H∞ Filter

The H∞ filter can be used to solve linear and state estimation problem. For the standard H∞ filter, trying to estimate the state $X \in R$ in the discrete time process, the equation can be expressed as [11, 12]

$$\begin{cases} X_k = \Phi_{k,k-1} \cdot X_{k-1} + W_{k-1} \\ Y_k = H_k \cdot X_k + V_k \end{cases} \tag{5}$$

where $\Phi_{k,k-1}$ is referred to as the state transition matrix, H_k is a matrix which relates to the state measurement. The random variables W_k and V_k represent the process noise and measurement noise (respectively). They are assumed to be independent, and with normal probability distributions $P(\omega)\sim(0,\ Q_k)$ and $P(\omega)\sim(0,\ R_k)$. In practice, the process noise covariance matrix Q_k and measurement noise covariance matrix R_k should be changed with each time measurement. Our goal is to estimate a linear combination of the state. So we estimate Z_k, which is given by

$$Z_k = L_k \cdot X_k. \tag{6}$$

If we want to directly estimate X_k, we will set user-defined matrix $L_k = I$. However, our goal is to estimate a linear combinations of the state. To find out the optimal state estimation $\widehat{Z_k}$, the equation is given by

$$J^* = \min_{\widehat{Z}_k} \max_{V_k, W_k, V_0} J. \tag{7}$$

3.2 Image Reconstruction Method Based on H∞ Filter

In ERT system, it supposes state-transition matrix $\Phi_{k,k-1}$ unit matrix, the measurement matrix H_k are sensitive field matrix, the measurement data $Y_k (0 \le k \le N-1)$ are the measured boundary voltages $V_k (0 \le k \le N-1)$, the estimated measurements values are pixel gray-values. ω_k and ν_k represent system noise sequence and measurement noise sequence. Therefore, the state space equation of image reconstruction is as follows:

$$\begin{cases} g_k = g_{k-1} + w_k \\ V_k = J \cdot g_k + v_k \\ Z_k = L_k \cdot g_k \end{cases} \tag{8}$$

To gain a smallest error estimate $e_k = Z_k - \widehat{Z_k}$, the H$\infty$ filter cost function is to gain each pixel. Therefore, the following equation is given by [13]

$$\theta_1 = sup \frac{\sum_{k=0}^{N-1} \|Z_k - \widehat{Z}_k\|^2_{S_k}}{\sum_{k=0}^{N-1}(\|w_k\|^2_{Q_k^{-1}} + \|v_k\|^2_{R_k^{-1}}) + \|g_0 - \widehat{g}_0\|^2_{P_0^{-1}}} \tag{9}$$

where, S_k, Q_k^{-1}, R_k^{-1} represent the weight matrixes of evaluated error, state error and measurement error, P_0 is defined the initial error value. According to the design criteria of H∞ filter, to find out the estimate $\widehat{Z_k}$, the equation about θ_1 is given by (r is called interference level)

$$\theta_1 < \frac{1}{r}. \tag{10}$$

So, Eq. (9) can be transformed into

$$\theta = -\frac{1}{r}\|g_0 - \widehat{g}_0\|^2_{P_0^{-1}} + \sum_{k=0}^{N-1}[\|Z_k - \widehat{Z}_k\|^2_{\overline{S_k}} - \frac{1}{r}(\|w_k\|^2_{Q_k^{-1}} + \|v_k - J \cdot g_k\|^2_{R_k^{-1}})] < 1. \tag{11}$$

From the type it can also be seen that, if the initial error values P_0 is high, and measurement accuracy is low. Meanwhile, when Q_k^{-1}, R_k^{-1} are bigger, θ is also lower. So when Q_k, R_k and P_0 are changed, θ can gain estimated minimize error in the unknown interfere condition. The problem is interpreted as

$$\theta^* = \min_{\widehat{g}_k} \max_{V_k, W_k, g_0} \theta. \tag{12}$$

The following condition must hold in order for the above estimator:

$$P^{-1}_{k/k-1} - r\overline{S_k} + J^T R_k^{-1} J > 0. \tag{13}$$

Since $\widehat{Z_k} = L_k \cdot \widehat{g_k}$, we see that $v_k = V_k - J \cdot g_k$ and

$$\|v_k\|^2_{R_k^{-1}} = \|v_k - J \cdot g_k\|^2_{R_k^{-1}}. \tag{14}$$

$$\|Z_k - \widehat{Z}_k\|^2_{S_k} = (g_k - \widehat{g}_k)^T L_k^T S_k L_k (g_k - \widehat{g}_k) = \|g_k - \widehat{g}_k\|^2_{\overline{S_k}} \tag{15}$$

$$\overline{S_k} = L_k^T S_k L_k. \tag{16}$$

We substitute these results to obtain

$$\begin{aligned} \theta &= -\frac{1}{r}\|g_0 - \widehat{g}_0\|^2_{P_0^{-1}} + \sum_{k=0}^{N-1}[\|Z_k - \widehat{Z}_k\|^2_{\overline{S_k}} - \frac{1}{r}(\|w_k\|^2_{Q_k^{-1}} + \|v_k - J \cdot g_k\|^2_{R_k^{-1}})] \\ &= \Psi(g_0) + \sum_{k=0}^{N-1} L_k. \end{aligned} \tag{17}$$

So the problem is transformed to solve extreme-value problem of various parameters, iterative equation about estimation problem can be formulates as

$$\widehat{g}_k = \widehat{g}_{k/k-1} + K_k(V_k - J \cdot \widehat{g}_{k/k-1}). \tag{18}$$

Filter gain matrix and error matrix can be expressed as

$$K_k = P_{k/k-1}(I - r\overline{S_k}P_{k/k-1} + J^T R_k^{-1} J P_{k/k-1})^{-1} J^T R_k^{-1} \tag{19}$$

$$P_{k/k-1} = P_{k/k-1}(I - r\overline{S_k}P_{k/k-1} + J^T R_k^{-1} J P_{k/k-1})^{-1} + Q_k \tag{20}$$

where k is the number of iterative steps. From Eq. (18), it only needs to compute filter gain matrix K_k that is directly related to the imaging result. So we can also see, the solution of filter gain matrix and error matrix result in burdensome calculation. And in dynamic system, disturbance level r will affect the estimated performance. At present, some H∞ filter can improve estimated result, but it is not adapted to the rapidly dynamic system. So, in order to meet the requirements of the online measurement, we assume that the initial parameters are close to the real distribution of information, and most of medium distribution features are identified after the first iteration. At the same time, reconstruction algorithm is considered to be linear in the dynamic imaging, which is called fast H∞ filter.

4 Result and Discussion

To verify the validity of the algorithm, the three typical distributions are selected to test algorithm. The data is obtained by using the 16-electrode ERT sensor. The low conductivity (background) and the high conductivity (objects) are 1 S/m and 2 S/m, respectively. The forward problem is solved by using a finite element method. A mesh of adaptive first-order triangular elements (about 576 elements) is produced in MATLAB. The measured voltages are given by the complete electrode model through adjacent driving patterns. And two evaluation criterions are used for quantitative comparison.

1. Correlation Coefficient-CC, between the test object and the reconstruction image, as defined in Eq. (21)

$$cc = \frac{\sum_{i=1}^{N}(\widehat{g}_i - \overline{\widehat{g}})(g_i - \overline{g})}{\sqrt{\sum_{i=1}^{N}(\widehat{g}_i - \overline{\widehat{g}})^2 \sum_{i=1}^{N}(g_i - \overline{g})^2}} \tag{21}$$

where g_i is the true permittivity distribution in the interested region, $\widehat{g}_i$ is the reconstructed permittivity distribution, and $\overline{g_i}$ and $\overline{\widehat{g}_i}$ are the mean values of g_i and $\widehat{g}_i$ respectively. The stronger correlation coefficient means the better image accuracy.

2. Root mean squared error, RMSE

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\widehat{g}_i - g_i)^2}. \tag{22}$$

Above equation illustrates approximation degree of original image and reconstruction image. If RMSE is enough small, it will gain the better image reconstruction results.

Fig. 2 shows the image reconstruction results under different interference revels. When r is bigger, gray scopes are increased. It results in that distribution is fragmented and image quality is significantly lower. Thus interference parameter has important influence to reconstructed results. The evaluation criterions (Correlation Coefficient and RMSE) based on three models (Bubble flow I, Bubble flow II and Stratified flow distributions in Fig. 2) are calculated and plotted in Fig. 3. As can be seen from Fig. 3, when r closes to 1, the image reconstruction results are diverging and unstable. When r is equal or lesser than 0.5, the reconstructed accuracy gains certain constant growth. Meanwhile, when the discrete distribution information in the soft field, image reconstruction quality is improved. But with the increasing of the medium discrete phase, result error is increased. Specially, in Fig. 3 (b) there are two large singular points when r is equal to 0.5 or 0.8, they have been normalized processing.

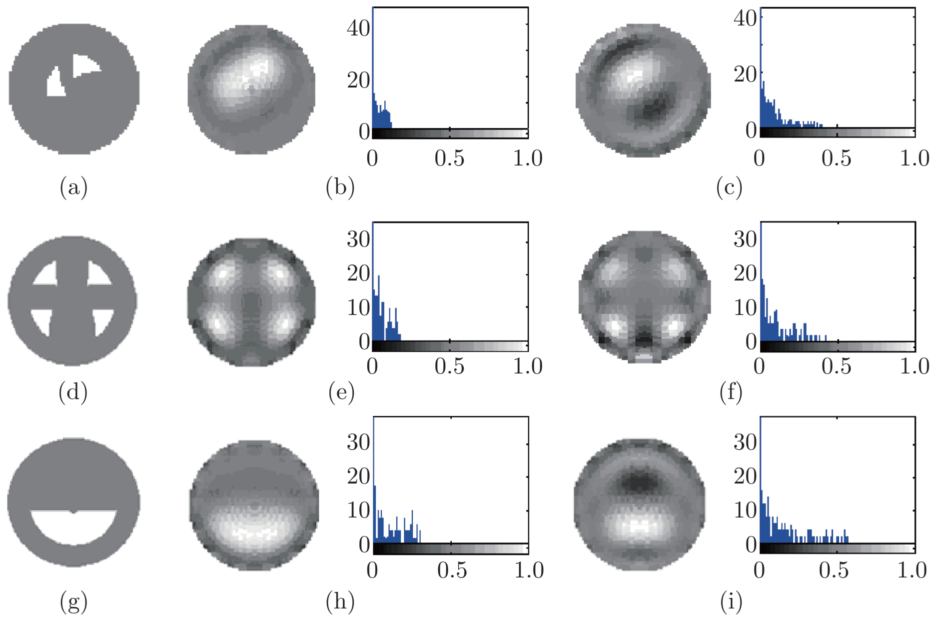

Fig. 2: Image reconstruction results and gray histogram by different model: (a), (d), (g) Bubble flow I, Bubble flow II and Stratified flow distributions; (b), (e), (h) Image reconstruction results and gray histogram when r is 0.1; (c), (f), (i) Image reconstruction results and gray histogram when r is 1

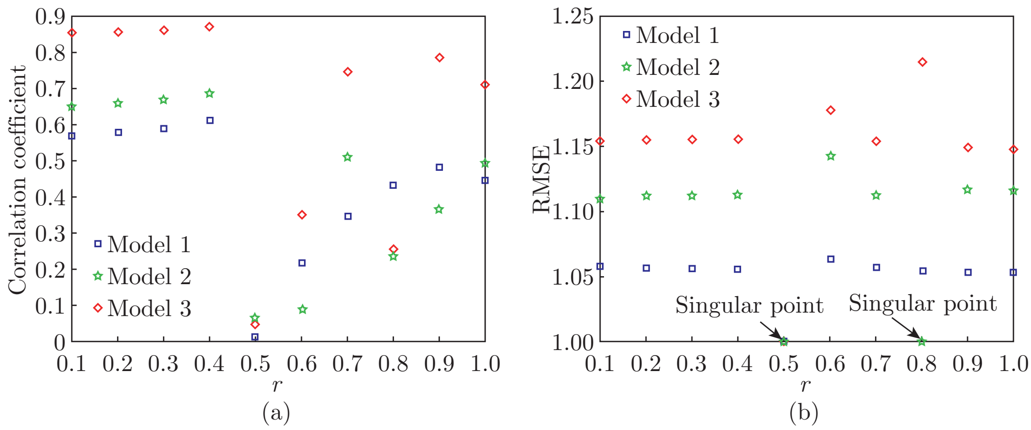

Fig. 3: The quality parameter distribution of different models. (a) Correlation Coefficient distribution. (b) RMSE distribution

In Table 1 and Table 2, the comparison results with Kalman filter in the three different distributions are given based on different system noises and measurement noises. We can also see that the fast H∞ filter method has the most robust with different noise information. In H∞ filter, we can see that subtracting $r\overline{S_k}P_k$ on the right side of the P_{k+1} equation tends to make P_{k+1} larger. Similarly, subtracting $r\overline{S_k}P_k$ on the right side of the K_k equation tends to make K_k larger. Comparing with the Kalman filter, the H∞ filter is a worst-case filter in the sense that it assumes that w_k, v_k $g_0 \in R^n$ will be chosen by nature to maximize the cost function. So, to solve the soft-filed problem and boundary disturbance noise in ERT, the robust feature of H∞ filter can be utilized to control the influence of measurement noise.

Table 1: Comparison of correlation coefficient and RMSE results when system noise covariation is 1 and r is 0.1

Method	Constraint	Model 1		Model 2		Model 3	
		CC	RMSE	CC	RMSE	CC	RMSE
H∞ filter	R_k=0.01	0.5694	1.0571	0.6501	1.1130	0.8532	1.1555
H∞ filter	R_k=1	0.5694	1.0571	0.6501	1.1130	0.8532	1.1555
Kalman filter	R_k=0.01	0.5694	1.0575	0.6427	1.1135	0.8497	1.1563
Kalman filter	R_k=1	0.3944	1.0751	0.3598	1.1422	0.6776	1.2082

Table 2: Comparison of correlation coefficient and RMSE results when measurement noise covariation is 0.01 and r is 0.01

Method	Constraint	Model 1		Model 2		Model 3	
		CC	RMSE	CC	RMSE	CC	RMSE
H∞ filter	Q_k=10	0.6175	1.0550	0.6894	1.1104	0.8736	1.1521
H∞ filter	Q_k=1	0.6175	1.0550	0.6894	1.1104	0.8736	1.1521
Kalman filter	Q_k=10	0.6139	1.0551	0.6874	1.1106	0.8724	1.1523
Kalman filter	Q_k=1	0.5624	1.0575	0.6427	1.1135	0.8497	1.1563

Computation time is another factor which must be considered in assessing the proposed method. Our numerical experiments show that the new method is more effective in convergence of algorithm. And it reduces the computation cost and computation time, comparing with the Gauss-Newton iterative (G-N) under 10 steps iteration and Kalman filter methods in Table 3.

Table 3: Comparison in terms of computational time

Methods	Computational time (s)		
	Two objects	Four objects	Stratified flow
G-N	3.6120	3.630	3.6570
Kalman filter	3.6627	3.4492	3.7158
Fast H∞ filter	0.7269	0.8893	0.8734

5 Conclusions

Aimed at the problem boundary noise interference on image reconstructed characteristics in ERT system, the paper proposes an image reconstruction algorithm based on fast one-step H∞ filter. It obtains the largest estimate error through calculating optimal filtering gain. Specially, in the case that the noise disturbance is uncertainty, it can gain more stable and robust reconstruction results. The algorithm has a certain loss on the reconstruction accuracy, but it improves the reconstruction speed. Consequently, H∞ filter method is fast and suitable for multi-phase flow on-line measurement.

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