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Nonclassical properties and generation of superposition state of excited coherent states of motion of trapped ion

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Abstract. We have studied the intensity squeezing of superposition state of excited coherent states and proposed a new method to prepare superposition state of excited coherent states of vibrational motion of trapped ion. This method is based on the interaction of a single trapped ion with two traveling wave light fields with different frequencies. An obvious merit of this method is that it works without application of the perturbation theory.

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Key words: trapped ion, excited coherent states, intensity squeezing

1 Introduction

In the past years, great efforts has been made to prepare a variety of nonclassical states of atoms and ion owing to their potential practical applications such as precision spectroscopy [1] and quantum computation [2]. So far many schemes have been proposed to prepare Fock states of the vibrational motion of the trapped ion, for example, by advantage of quantum jump effects [3], or by applying two-color laser fields successively [4]. Squeezed states of motion of trapped ion can be generated by applying two standing wave laser fields with different frequency [5], or by Raman transition in two trapped ions [6]. The squeezed states of light fields can be prepared in high-Q cavity [7,8]. The entanglement of coherent motional states of multiple trapped ions can be generated [9,10]. In addition, the authors of Ref. [11] have investigated the excitation coherent state and proposed a scheme for its preparation based on perturbation theory. The authors of Ref. [12] investigated nonclassical properties of states generated by the superposition of excitation coherent state. They found that effect of the phase of coherence states on the evolution of mean photon number and squeezed properties. The authors of Ref. [13] investigated Wigner function for the photon-added even

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and odd coherent state. Schemes for generation of these states have been proposed, but they are all based on perturbation theory.

In this paper, we investigate the intensity squeezing of excited Schrödinger cat states and propose a method to prepare the excited Schrödinger cat states of vibrational motion of trapped ion. This method is based on interaction of a single trapped ion with carrier and blue side traveling wave light fields. The excited coherent state of the motion of the ion can be produced by controlling the interaction time of the ion with fields.

2 Intensity squeezing of excited Schrödinger cat states

The excited Schrödinger cat states is defined as

$$\left|\varphi^{(m)}\right\rangle_{\pm}=\frac{1}{\sqrt{N}}(a^{+})^{m}\Big(|\alpha\rangle+e^{i\phi}|\alpha\rangle\Big), \tag{1a}$$

$$N=2m!\Big(L_m(-|\alpha|^2)+\cos\phi L_m(|\alpha|^2)e^{-2|\alpha|^2}\Big), \tag{1b}$$

where N is a normalized constant, $L_m(-x)$ is Laguerre polynomial, $\alpha=|\alpha|e^{i\theta}=re^{i\theta}$. Here we study intensity squeezing of the excited Schrödinger cat states. For this goal we define orthogonal Hermite operators X_1 and X_2 as

$$X_1=\frac{1}{2}(a^2+a^{+2}),\qquad X_2=\frac{1}{2}(a^{+2}-a^2). \tag{2}$$

The fluctuation of the operators X_i satisfies

$$\Delta X_1\Delta X_2\geq\frac{|\langle C\rangle|}{2}, \tag{3a}$$

where

$$\Delta X_i=\sqrt{\langle X_i^2\rangle-\langle X_i\rangle^2},\qquad C=[X,Y]. \tag{3b}$$

If the fluctuation of the operator X_i in a state satisfies the relation

$$S_i=\frac{\Delta X_i-\big|\langle C\rangle\big|}{2}<0, \tag{4}$$

we then say this state has the property of intensity squeezing. Using the formulae

$$a^n(a^{+})^m=\sum_{k=0}^{m}\frac{m!n!(a^{+})^k a^{k+n-m}}{(m-n)!(n-m+k)!k!}, \tag{5}$$

and expression of the associated Laguerre polynomial

$$L_m^{(l)}(-x)=\sum_{n=0}^{m}\frac{(m+l)!}{(m-n)!n!(l+n)!}(-x)^n, \tag{6}$$

we easily compute the squeezing quantity of the operators X_1 in the excited Schrödinger cat state, which is represented as

$$
\begin{aligned}
S(m,\alpha,\phi)=&\Delta X_1-\frac{\langle C\rangle}{2}\\
=&\Bigg(\frac{m!}{N}\Big((m+2)(m+1)L_m(-|\alpha|^2)+(m+2)(m+1)\cos(\phi)e^{-2|\alpha|^2}L_m(|\alpha|^2)\\
&-2(m+1)L_{m+1}(|\alpha|^2)-2(m+1)\cos(\phi)e^{-2|\alpha|^2}L_{m+1}(|\alpha|^2)\\
&+\frac{1}{2}(\alpha^4+\alpha^{*4})\Big(L_m^{(4)}(-|\alpha|^2)+\cos(\phi)L_m^{(4)}(|\alpha|^2|)\Big)+\frac{4N}{m!}\\
&-\frac{m!}{N}(\alpha^2+\alpha^{*2})^2\Big(L_m^{(2)}(-|\alpha|^2)+\cos(\phi)e^{2|\alpha|^2}L_m^{(2)}(|\alpha|^2)\Big)^2\Big)\Bigg)^{1/2}\\
&-\frac{4(m+1)!}{N}\Big(L_{m+1}(-|\alpha|^2)+\cos(\phi)e^{-2|\alpha|^2}L_{m+1}(|\alpha|^2)\Big)+1.
\end{aligned}
\tag{7}
$$

In the following numerical computation, for simplicity we select $\theta=0$. Fig. 1(a) shows evolution of the squeezing quantity of excited odd coherent states with mean photon number r with different m. The lager the parameters m and r are, the deeper the squeezing of this state is. For evolution of the squeezing quantity of the excited even coherent states, the same properties also occurs, which is shown in Fig. 1(b). But, unlike excited odd coherent states, the excited even coherent states dos not show squeezing when the parameters m and r are small (see Fig. 1(b)).

3 Preparation of excited coherent states

The excited coherent states is defined as [11]

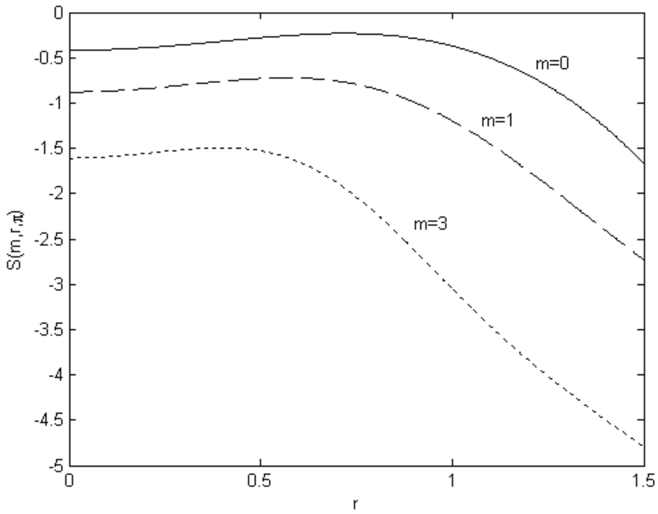
$$
|\Phi\rangle=\frac{1}{\sqrt{|\alpha|^2+1}}a^\dagger|\alpha\rangle. \tag{8}
$$

With application of displacement operator and its properties, we obtain

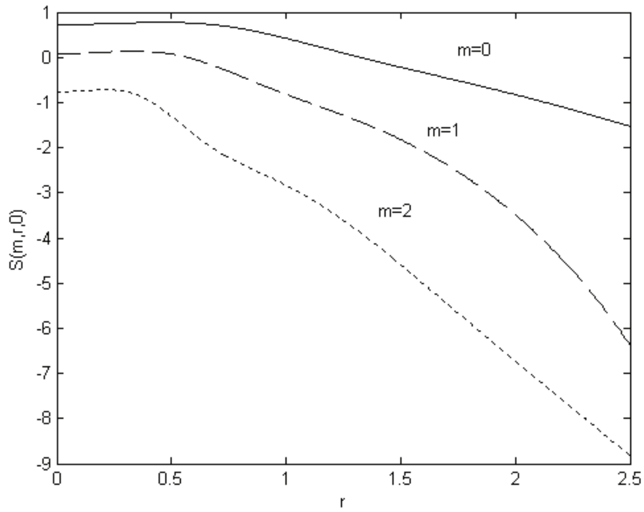
$$
\begin{aligned}
|\Phi\rangle=&\frac{1}{\sqrt{|\alpha|^2+1}}a^\dagger D(\alpha)|0\rangle\\
=&\frac{1}{\sqrt{|\alpha|^2+1}}D(\alpha)\Big(|1\rangle+\alpha^*|0\rangle\Big).
\end{aligned}
\tag{9}
$$

The excitation coherent states exhibit nonclassical properties such as sub-Poissonian photon statistics [11].

Here we propose a new scheme for generation of the excited coherent states. Let's consider a two-level trapped ion with energy difference $\hbar\omega_a$, which interacts with two traveling wave



(a)



(b)

Figure 1: The evolution of the squeezing quantity $S(m,r,\phi)$ via the parameter r for different parameter m,ϕ, (a) $\phi=\pi$, (b) $\phi=0$.

light fields with frequency ω_L, $\omega_L-\omega$, respectively, ω is the vibrational frequency of the ion. After rotating wave approximation, the Hamiltonian of the system can be written as ($\hbar=1$)

$$H=\omega a^+a+\frac{\delta}{2}\sigma_Z+\left(\frac{\Omega_1}{2}e^{i\eta(a+a^+)}\sigma_++\frac{\Omega_2}{2}e^{i\eta(a+a^+)+i\omega t}\sigma_++H.c.\right), \tag{10}$$

where $\delta=\omega_a-\omega_L$, a^+ and a are the creation and annihilation operators of motion of the ion, σ_Z and $\sigma_\pm$ are Pauli's operators, $\sigma_z=|e\rangle\langle e|-|g\rangle\langle g|$ and $\sigma_+=|e\rangle\langle g|$, $\sigma_-=|g\rangle\langle e|$, and

$\Omega_{1,2}$ is the coupling constant of traveling wave light fields. With application of the Lamb-Dicke approximation, we can obtain the interaction Hamiltonian in the interaction picture as follows

$$H_I' = \frac{\delta}{2}\sigma_Z + ig(a\sigma_+ - a^+\sigma_-) + \frac{\Omega_1}{2}\sigma_x, \tag{11}$$

where $\sigma_x = \sigma_+ + \sigma_- g = i\eta\Omega_2/2$. The dynamics of the anti-JC model corresponding to the Hamiltonian (11) can be solved exactly. For this we make a unitary transformation on Eq. 11, $H"_I = D^+(i\alpha)H_I'D(i\alpha)$, $D(i\alpha) = e^{i\alpha(a^+ + a)}$, $\alpha = \Omega_1/2\Omega_2$, the interaction Hamiltonian can be represented as

$$H_I'' = \frac{\delta}{2}\sigma_Z + ig(a\sigma_+ - a^+\sigma_-). \tag{12}$$

The unitary evolution operator corresponding to the interaction Hamiltonian (12) reads

$$U_1' = U_{11}'|e\rangle\langle e| + U_{12}'|e\rangle\langle g| + U_{21}'|g\rangle\langle e| + U_{22}'|g\rangle\langle g|, \tag{13}$$

where

$$U_{11}'(t) = \cos(K_{n+1}t) - i\delta\frac{\sin(K_{n+1}t)}{2K_{n+1}}, \tag{14}$$

$$U_{12}'(t) = ga\frac{\sin(K_n t)}{K_n}, \tag{15}$$

$$U_{21}'(t) = -ga^+\frac{\sin(K_{n+1}t)}{K_{n+1}}, \tag{16}$$

$$U_{22}'(t) = \cos(K_n t) + i\delta\frac{\sin(K_n t)}{2K_n}, \tag{17}$$

$$K_n = \sqrt{\frac{\delta}{4} + g^2 a^+ a}. \tag{18}$$

We now proceed to give a solution to the Hamiltonian (11), which reads

$$|\psi(t)\rangle = U_I|\psi(0)\rangle, \tag{19}$$

where $|\psi(0)\rangle$ is the initial wave vector

$$U_I = D(i\alpha)U_I'D^+(i\alpha). \tag{20}$$

Now we assume that at initial time the atom is located in the ground state $|g\rangle$ and cavity field prepared in the coherent state $|i\alpha\rangle$. After interaction time τ, we can obtain the state vector of the system as

$$\begin{aligned}|\psi(\tau)\rangle &= \frac{1}{\sqrt{2}}D(i\alpha)\Big(\big(U_{11}'(\tau) + U_{12}'(\tau)\big)|e\rangle + \big(U_{21}'(\tau) + U_{22}'(\tau)\big)|g\rangle\Big)|0\rangle \\ &= \frac{1}{\sqrt{2}}D(i\alpha)\left(\cos(K_1\tau) - i\delta\frac{\sin(K_1\tau)}{2K_1}\right)|0\rangle|e\rangle \\ &\quad + \frac{1}{\sqrt{2}}D(i\alpha)\left(-g\frac{\sin(K_1\tau)}{K_1}|1\rangle + e^{i\delta\tau/2}|0\rangle\right)|g\rangle,\end{aligned} \tag{21}$$

where $K_1 = \sqrt{\delta^2/4+g^2}$. If detection on the ground state $|g\rangle$ of the atom is done, we then obtain

$$|\psi_c\rangle = -\frac{g\sin(K_1\tau)}{\sqrt{2}K_1} D(i\alpha)\left(|1\rangle - \frac{K_1}{g\sin(K_1\tau)} e^{i\delta\tau/2}|0\rangle\right)|g\rangle. \tag{22}$$

Here we select the interaction time τ and the detuning parameter g, δ to satisfy the equations

$$\delta\tau = 3\pi, \tag{23}$$

$$\frac{K_1}{g\sin(K_1\tau)} = \alpha. \tag{24}$$

We can obtain

$$|\psi_{c1}\rangle = -\frac{g}{\sqrt{2}K_1}\sin(K_1\tau) D(i\alpha)\Big(|1\rangle - i\alpha|0\rangle\Big). \tag{25}$$

After operation of normalization, we have

$$|\psi_{c1}\rangle = \frac{1}{\sqrt{N}} D(i\alpha)\Big(|1\rangle - i\alpha|0\rangle\Big). \tag{26}$$

From Eq. 26 we can see that the atom is prepared in the excitation coherent state. If we detect on the excitation state $|e\rangle$ of the atom, we then obtain

$$|\psi_{c2}\rangle = \frac{1}{\sqrt{2}}\left(\cos(K_1\tau) - i\frac{\delta\sin(K_1\tau)}{2K_1}\right) D(i\alpha)|0\rangle. \tag{27}$$

This is a coherent state, not an excitation coherent state. Therefore the success probability for this scheme is 50%.

4 Preparation of excited Schr odinger cat states

Here we give a procedure to prepare excited Schrödinger cat states. The excited Schrödinger cat states is defined as

$$\begin{aligned}|\varphi\rangle_\pm &= \frac{1}{\sqrt{N}}\Big(|\alpha\rangle \pm |-\alpha\rangle\Big) \\ &= \frac{1}{\sqrt{N}}\left(\Big(D^+(\alpha) \pm D(\alpha)\Big)|1\rangle + \alpha^*\Big(D(\alpha) \mp D^+(\alpha)\Big)|0\rangle\right),\end{aligned} \tag{28}$$

where N is normalized constant. The preparation procedure for this state contains the following steps:

(i) We assume the initial state of vibrational motion of the trapped ion has been prepared in excitation coherent state, and the initial state of electric motion of the ion is the ground state $|g\rangle$, namely

$$|\psi(0)\rangle = \frac{1}{\sqrt{N}} D(i\alpha)\Big(i\alpha|0\rangle\Big)|g\rangle. \tag{29}$$

(ii) Applying the carrier frequency $\pi/2$ light pulse to excite the trapped ion, we can obtain the state vector of the system as

$$\begin{aligned}|\psi_1\rangle &= e^{i\pi\sigma_x/4}|\psi(0)\rangle \\ &= \frac{1}{\sqrt{2N}}D(i\alpha)\Big(|1\rangle - i\alpha|0\rangle\Big)|g\rangle + \frac{i}{\sqrt{2N}}D(i\alpha)\Big(i\alpha|0\rangle\Big)|e\rangle. \end{aligned} \tag{30}$$

(iii) Applying a $\pi/4$ light pulse to excite dispersively the trapped ion with the evolution operation $U_2 = e^{i\pi a^+ a\sigma_Z/2}$, we can obtain the state vector of the system as

$$|\psi_2\rangle = U_2|\psi_1\rangle \tag{31}$$

$$= -\frac{i}{\sqrt{2N}}D(\alpha)\Big(|1\rangle + \alpha|0\rangle\Big)|g\rangle - \frac{1}{\sqrt{2N}}D(-\alpha)\Big(|1\rangle - \alpha|0\rangle\Big)|e\rangle. \tag{32}$$

(iv) Further applying the carries frequency $\pi/2$ light pulse to excite the trapped ion, we can obtain the state vector of the system as

$$\begin{aligned}|\psi_3\rangle &= e^{i\pi\sigma_x/4}|\psi_2\rangle \\ &= -\frac{i}{\sqrt{2N}}\Big(D(-\alpha)\Big(|1\rangle - \alpha|0\rangle\Big) + D(\alpha)\Big(|\alpha\rangle + \alpha|0\rangle\Big)\Big)|g\rangle \\ &\quad + \frac{1}{\sqrt{2N}}\Big(D(-\alpha)\Big(|1\rangle - \alpha|0\rangle\Big) - D(\alpha)\Big(|1\rangle + \alpha|0\rangle\Big)\Big)|e\rangle \\ &= -\frac{i}{\sqrt{2N}}|\varphi\rangle_+|g\rangle + \frac{1}{\sqrt{2N}}|\varphi\rangle_-|e\rangle. \end{aligned} \tag{33}$$

(v) The measurement on the trapped ion is made. If detection result is that the ion is located in the ground state $|g\rangle$, then the ion can prepared in the $|\varphi\rangle_+$. If detection result is that the ion is in the excitation state $|e\rangle$, then the ion can prepared in the $|\varphi\rangle_-$.

In conclusion, we have investigated theoretically a scheme to prepare excitation coherent states of the trapped ion and its superposition states. This scheme is based on interaction of the trapped ion with classical traveling wave light fields.

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