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A FIFTH-ORDER ACCURATE WEIGHTED ENN DIFFERENCE SCHEME AND ITS APPLICATIONS*1)

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Abstract

In this paper, we have constructed a high accurate difference scheme based on the ENN scheme [1]. The new scheme has 5th-order accuracy in smooth regions and can keep the essencially non-oscillatory property.

Key words: ENN scheme, WENO scheme, Shock-boundary-layer interaction.

1. Introduction

In the paper [1] , where Zhang Hanxin et al. presented the nonoscillatory 3rd-order ENN difference scheme. The idea of ENN scheme is to compare the 1st-order difference and 2nd-order difference to attain 3rd-order accurate scheme and to avoid spurious oscillations near shocks. However the ENN scheme has certain drawbacks. One problem is only 3rd-order accuracy even in the very smooth regions. Another is to use a lot of logical statements which affect the convergence rate and the efficiency of parallel computing.

Recently, G.-S. Jiang and C.-W. Shu developed a 5th-order weighted ENO scheme [2] based on the third-order accurate difference scheme in the flux form. We found that the third-order accurate ENO scheme given in [2] with r=3 is the same as the ENN scheme without the limiters. Naturally, the ENN scheme would be expanded to 5th-order accurate scheme by using the idea of deriving the 5th-order WENO difference scheme.

In this paper, we have constructed the higher accuracy difference scheme based on the ENN scheme. The new scheme has 5th-order accuracy in smooth regions and can keep the essencially non-oscillatory property.

We tested the new scheme's accuracy by using a linear initial problem and tested its non-oscillatory property by using a nonlinear initial problem. At last, we applied the new scheme to compute the problem of shock-boundary-layer interaction. Numerical results showed that the new scheme is efficient.

2. ENN Scheme and Several High Order Accuracy Central Schemes

Consider a scalar conservative hyperbolic equation

$$\frac{\partial u}{\partial t} + \frac{\partial f(u)}{\partial x} = 0 \tag{1}$$

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where f is a flux function and can be splitted into two parts, i.e. $f(u) = f^+(u) + f^-(u)$ and $df^+(u)/du \geq 0$ *and* $df^-(u)/du \leq 0$. In this paper we define $f^\pm(u) = \frac{1}{2}(f(u) \pm \alpha u)$ and $\alpha = max|f'(u)|$ for one-dimensional equation. The semi-discrete conservative difference scheme can be written as follows

$$\frac{du_j}{dt} + \frac{(h_{j+\frac{1}{2}} - h_{j-\frac{1}{2}})}{\Delta x} = 0 \tag{2}$$

where the numerical flux $h_{j+\frac{1}{2}} = h^+_{j+\frac{1}{2}} + h^-_{j+\frac{1}{2}}$.

(1) The ENN scheme[1]

$$h^+_{j+\frac{1}{2}} = \begin{cases} f^+_j + \frac{1}{2}\Delta f^+_{j+\frac{1}{2}} - \frac{1}{6}ms(D^+_j, D^+_{j+1}) & if \mid \Delta f^+_{j+\frac{1}{2}} \mid\leq\mid \Delta f^+_{j-\frac{1}{2}} \mid & (3) \\ f^+_j + \frac{1}{2}\Delta f^+_{j-\frac{1}{2}} + \frac{1}{3}ms(D^+_j, D^+_{j-1}) & if \mid \Delta f^+_{j+\frac{1}{2}} \mid>\mid \Delta f^+_{j-\frac{1}{2}} \mid & (4) \end{cases}$$

$$h^-_{j+\frac{1}{2}} = \begin{cases} f^-_{j+1} - \frac{1}{2}\Delta f^-_{j+\frac{3}{2}} + \frac{1}{3}ms(D^-_{j+1}, D^-_{j+2}) & if \mid \Delta f^-_{j+\frac{3}{2}} \mid\leq\mid \Delta f^-_{j+\frac{1}{2}} \mid & (5) \\ f^-_{j+1} - \frac{1}{2}\Delta f^-_{j+\frac{1}{2}} - \frac{1}{6}ms(D^-_j, D^-_{j+1}) & if \mid \Delta f^-_{j+\frac{3}{2}} \mid>\mid \Delta f^-_{j+\frac{1}{2}} \mid & (6) \end{cases}$$

where $D_j = \Delta f_{j+\frac{1}{2}} - \Delta f_{j-\frac{1}{2}}$ and the ms(a,b) is defined below

$$ms(a,b) = \begin{cases} a & \mid a \mid\leq\mid b \mid \\ b & \mid a \mid>\mid b \mid \end{cases} \tag{7}$$

(2) 4th-order accurate central schemes [3]

$$h^+_{j+\frac{1}{2}} = f^+_j + \frac{1}{2}\Delta f^+_{j+\frac{1}{2}} - \frac{1}{12}D^+_j - \frac{1}{12}D^+_{j+1} \tag{8}$$

$$h^+_{j+\frac{1}{2}} = f^+_j + \frac{1}{2}\Delta f^+_{j-\frac{1}{2}} + \frac{1}{4}D^+_j + \frac{1}{12}D^+_{j-1} \tag{9}$$

$$h^-_{j+\frac{1}{2}} = f^-_{j+1} - \frac{1}{2}\Delta f^-_{j+\frac{3}{2}} + \frac{1}{4}D^-_{j+1} + \frac{1}{12}D^-_{j+2} \tag{10}$$

$$h^-_{j+\frac{1}{2}} = f^-_{j+1} - \frac{1}{2}\Delta f^-_{j+\frac{1}{2}} - \frac{1}{12}D^-_{j+1} - \frac{1}{12}D^-_j \tag{11}$$

(3) 5-order accurate central scheme

$$h^+_{j+\frac{1}{2}} = f^+_j + \frac{3}{5} \times (\frac{1}{2}\Delta f^+_{j+\frac{1}{2}} - \frac{1}{12}D^+_j - \frac{1}{12}D^+_{j+1}) + \frac{2}{5} \times (\frac{1}{2}\Delta f^+_{j-\frac{1}{2}} + \frac{1}{4}D^+_j + \frac{1}{12}D^+_{j-1}) \tag{12}$$

$$h^-_{j+\frac{1}{2}} = f^-_{j+1} + \frac{2}{5} \times (-\frac{1}{2}\Delta f^-_{j+\frac{3}{2}} + \frac{1}{4}D^-_{j+1} + \frac{1}{12}D^-_{j+2}) + \frac{3}{5} \times (-\frac{1}{2}\Delta f^-_{j+\frac{1}{2}} - \frac{1}{12}D^-_{j+1} - \frac{1}{12}D^-_j) \tag{13}$$

3. Numerical Method

For simplicity, we show only the positive part of the splitted flux, and the negative part of the splitted flux are symmetric with respect to $x_{j+\frac{1}{2}}$.

From above equations, it can be seen that the combinal coefficients in (8) from (3) are

$$C^1_1 = \frac{1}{2}, C^1_2 = \frac{1}{2};$$

the combinal coefficients in (9) from (4) are

$$C^2_1 = \frac{3}{4}, C^2_2 = \frac{1}{4};$$

and in (12) from (8) & (9),the coefficients are

$$C_1=\frac{3}{5},C_2=\frac{2}{5}.$$

(1) 4th-order weighted scheme

First, we weight (3) to obtain two 4th-order accurate numerical fluxes:

$$h^{1+}_{j+\frac{1}{2}}=f^+_j+\frac{1}{2}\Delta f^+_{j+\frac{1}{2}}-\frac{1}{6}(\omega^+_1 D^+_j+\omega^+_2 D^+_{j+1}) \tag{14}$$

$$h^{2+}_{j+\frac{1}{2}}=f^+_j+\frac{1}{2}\Delta f^+_{j-\frac{1}{2}}+\frac{1}{3}(\omega^+_3 D^+_j+\omega^+_4 D^+_{j-1}) \tag{15}$$

where

$$\omega^+_1=\frac{\alpha_1}{\alpha_1+\alpha_2},\omega^+_2=\frac{\alpha_2}{\alpha_1+\alpha_2},\omega^+_3=\frac{\alpha_3}{\alpha_3+\alpha_4},\omega^+_4=\frac{\alpha_4}{\alpha_3+\alpha_4}.$$

$$\alpha_1=\frac{C^1_1}{(\varepsilon+W_1)},\alpha_2=\frac{C^1_2}{(\varepsilon+W_2)},\alpha_3=\frac{C^2_1}{(\varepsilon+W_3)},\alpha_4=\frac{C^2_2}{(\varepsilon+W_4)}, \tag{16}$$

Based on the idea of the ENN scheme, we can obtain the 4th-order non-oscillatory scheme:

$$h^{4+}_{j+\frac{1}{2}}=\begin{cases} f^+_j+\frac{1}{2}\Delta f^+_{j+\frac{1}{2}}-\frac{1}{6}(\omega^+_1 D^+_j+\omega^+_2 D^+_{j+1}) \\ \quad if \quad \mid \Delta f^+_{j+\frac{1}{2}}\mid\le\mid \Delta f^+_{j-\frac{1}{2}}\mid \\ f^+_j+\frac{1}{2}\Delta f^+_{j-\frac{1}{2}}+\frac{1}{3}(\omega^+_3 D^+_j+\omega^+_4 D^+_{j-1}) \\ \quad if \quad \mid \Delta f^+_{j+\frac{1}{2}}\mid>\mid \Delta f^+_{j-\frac{1}{2}}\mid \end{cases} \tag{17}$$

Through theoretical analysis and numerical result, the scheme (17) is 4th-order accurate in smooth regions, but only 3th-order in critical points.

The variables $W_k(k=1,...,4)$ will be discussed in section 4.

(2) 5th-order weighted scheme

We suppose that (14) and (15) are two 4th-order numerical fluxes, then we construct the 5th-order accurate scheme as follows:

$$h^+_{j+\frac{1}{2}}=\Omega^+_1 h^{1+}_{j+\frac{1}{2}}+\Omega^+_2 h^{2+}_{j+\frac{1}{2}} \tag{18}$$

where

$$\Omega^+_1=\frac{\beta_1}{\beta_1+\beta_2},\Omega^+_2=\frac{\beta_2}{\beta_1+\beta_2},$$

$$\beta_1=\frac{C_1}{(\varepsilon+W^1_5)},\beta_2=\frac{C_2}{(\varepsilon+W^2_5)}, \tag{19}$$

The variables $W^k_5(k=1,2)$ will be discussed in section 4.

4. The Relation between the Weighted Functions and the Accuracy of Scheme

To achieve essentially non-oscillatory property, we require the scheme (18) approachs to the scheme (17) near shocks, on the other hand, we requre the scheme (18) approachs to 5th-order accurate scheme (12) in smooth regions. Therefore, the weights must assure that only one flux is played into in discontinuous regions, and all fluxes go into effect in smooth regions.

We suppose $h^{n,1}_{j+\frac{1}{2}}$ *and* $h^{n,2}_{j+\frac{1}{2}}$ are two nth-order accurate flux functions, $h^{n+1}_{j+\frac{1}{2}}$ is a (n+1)th order accurate flux function, which is expressed through the following relation

$$h^{n+1}_{j+\frac{1}{2}} = C_1 h^{n,1}_{j+\frac{1}{2}} + C_2 h^{n,2}_{j+\frac{1}{2}}, C_1, C_2 > 0, C_1 + C_2 = 1,$$

where, we require $\omega_1, \omega_2 > 0, \omega_1 + \omega_2 = 1$, and

$$\sum_{i=1}^{2} \omega_i h^{n,i}_{j+\frac{1}{2}} = h^{n+1}_{j+\frac{1}{2}} + \sum_{i=1}^{2} (\omega_i - C_i)(h^{n,i}_{j+\frac{1}{2}} - h^{n+1}_{j+\frac{1}{2}}) \tag{20}$$

If ω_i satisfies

$$\omega_i = C_i + O(h) \tag{21}$$

then the convex combination $\sum_{i=1}^{2} \omega_i h^{n,i}_{j+\frac{1}{2}}$ is the (n+1)th order accurate flux function. Of course, if $\omega_i = C_i + O(h^l), l > 1$,the scheme is more approximate to the (n+1)th order scheme.

In the paper [2], authors presented a smoothness measurement :

$$IS_k = \sum_{i=1}^{r-1} \int_{x_{j-\frac{1}{2}}}^{x_{j+\frac{1}{2}}} h^{2l-1} (q_k^{(l)})^2 dx, k = 0, ..., r-1 \tag{22}$$

where $q_k(x)$ is the interpolation polynomial on the stencil $(x_{j+k-r+1}, ..., x_{j+k}), q_k^{(l)}$ is the l^{th} derivative of $q_k(x)$.

For r=3, the (22) gives

$$\begin{aligned} IS_0 &= \tfrac{13}{12}(f_{j-2} - 2f_{j-1} + f_j)^2 + \tfrac{1}{4}(f_{j-2} - 4f_{j-1} + 3f_j)^2 \\ IS_1 &= \tfrac{13}{12}(f_{j-1} - 2f_j + f_{-1})^2 + \tfrac{1}{4}(f_{j-1} - f_{j+1})^2 \\ IS_2 &= \tfrac{13}{12}(f_j - 2f_{j+1} + f_{j+2})^2 + \tfrac{1}{4}(3f_j - 4f_{j+1} + f_{j+2})^2 \end{aligned} \tag{23}$$

In smooth regions, Taylor expansions of (23) give, respectively

$$\begin{aligned} IS_0 &= \tfrac{13}{12}(f^{''} h^2 - f^{'''} h^3)^2 + \tfrac{1}{4}(2f^{'} h - \tfrac{2}{3} f^{'''} h^3)^2 + O(h^6) \\ IS_1 &= \tfrac{13}{12}(f^{''} h^2)^2 + \tfrac{1}{4}(2f^{'} h + \tfrac{1}{3} f^{'''} h^3)^2 + O(h^6) \\ IS_2 &= \tfrac{13}{12}(f^{''} h^2 + f^{'''} h^3)^2 + \tfrac{1}{4}(2f^{'} h - \tfrac{2}{3} f^{'''} h^3)^2 + O(h^6) \end{aligned} \tag{24}$$

If $f^{'} \neq 0$, then

$$IS_k = (f^{'} h)^2 (1 + O(h^2)), k = 0, 1, 2. \tag{25}$$

When $f^{'} = 0$, then

$$IS_k = \frac{13}{12} (f^{''} h^2)^2 (1 + O(h)), k = 0, 1, 2 \tag{26}$$

Therefore, in (16), we take

$$W_1 = IS_1, W_2 = IS_2, W_3 = IS_1, W_4 = IS_0 \tag{27}$$

and in (19), we take

$$W_5^1 = IS_2, W_5^2 = IS_0 \tag{28}$$

these variables will satisfy condition (21).

In other way, we can find new weight functions through the linear extrapolating by using the 1st-order differences $\Delta f_{j-\frac{3}{2}}, \Delta f_{j-\frac{1}{2}}, and \Delta f_{j+\frac{1}{2}}, \Delta f_{j+\frac{3}{2}}$,i.e. in (19) take

$$W_5^1 = \frac{1}{2}(3\Delta f^+_{j+\frac{1}{2}} - \Delta f^+_{j+\frac{3}{2}})^2, W_5^2 = \frac{1}{2}(3\Delta f^+_{j-\frac{1}{2}} - \Delta f^+_{j-\frac{3}{2}})^2 \tag{29}$$

In smooth regions, Taylor expansions of (29) give

$$W_5^1 = (f^{'}h - \frac{1}{2}f^{'''}h^3 + O(h^4))^2$$

$$W_5^2 = (f^{'}h - \frac{1}{2}f^{'''}h^3 + O(h^4))^2 \tag{30}$$

If $f^{'} \neq 0$, then

$$W_5^k = (f^{'})^2h^2(1 + O(h^2)), k = 0, 1, \tag{31}$$

When $f^{'} = 0$, then

$$W_5^k = \frac{1}{4}(f^{'''})^2h^6(1 + O(h)), k = 0, 1 \tag{32}$$

So they can also satisfy the condition (21) and have the same accuracy.

Variable ε is a positive real number which is introduced to avoid the denominator to become zero, in this paper, $\varepsilon = 10^{-6}$.

5. Numerical Examples

(1) the linear initial problem

$$u_t + u_x = 0, -1 \leq x \leq 1$$

$$u(x, 0) = u_0(x) \qquad periodic. \tag{33}$$

In Table 1, we show the errors of these schemes at t=1 for the initial condition $u_0(x) = sin(\pi x)$.

In Table 2, we show the errors of these schemes at t=1 for the initial condition $u_0(x) = sin^4(\pi x)$.

(2) the nonlinear initial problem

$$\frac{\partial u}{\partial t} + \frac{\partial(\frac{u^2}{2})}{\partial x} = 0, \qquad 0 \leq x \leq 2\pi$$

$$u(x, 0) = 0.3 + 0.7\mathrm{sin}x, \qquad 0 \leq x \leq 2\pi \tag{34}$$

The numerical results can be seen in fig. 1 with N=40 and fig. 2 with N=80 for $t = 2$.

(3) the nonlinear Burgers equation

$$\begin{aligned} &\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{1}{Re}\frac{\partial^2 u}{\partial x^2} \\ &u(-1, t) = 1, u(1, t) = 1, u(x, 0) = 0, -1 < x < 1. \end{aligned} \tag{35}$$

We computed the steady solution of (35) with $Re = 1000$, in Table 3, given the convergent times and errors of several schemes, where convergent criterion is:

$$\frac{1}{N}\Sigma_{i=1}^N|u_i^{n+1} - u_i^n|/\Delta t \leq 0.5 \times 10^{-3}$$

.

Table 1: Accuracy on $u_t + u_x = 0$, with $u_0(x) = sin(\pi x)$.

Method	N	$L_\infty error$	$L_\infty order$	$L_1 error$	$L_1 order$
ENN	10	6.048e-2	—–	3.753e-2	—–
	20	8.606e-3	2.813	4.917e-3	2.932
	40	1.076e-3	3.000	6.307e-4	2.963
	80	1.353e-4	2.991	7.977e-5	2.983
	160	1.690e-5	3.001	1.003e-5	2.992
	320	2.094e-6	3.013	1.258e-6	2.995
WENN-5	10	1.763e-2	—–	1.005e-2	—–
	20	8.908e-4	4.307	4.323e-4	4.539
	40	2.711e-5	5.038	1.283e-5	5.074
	80	8.733e-7	4.956	3.950e-7	5.022
	160	2.549e-8	5.099	1.238e-8	4.996
	320	7.270e-10	5.132	3.872e-10	4.998
WENO-5	10	2.958e-2	—–	1.591e-2	—–
	20	1.455e-3	4.345	7.388e-4	4.429
	40	4.591e-5	4.986	2.221e-5	5.056
	80	1.475e-6	4.960	6.904e-7	5.008
	160	4.359e-8	5.081	2.166e-8	4.994
	320	1.277e-9	5.093	6.774e-10	4.998

Table 2: Accuracy on $u_t + u_x = 0$, with $u_0(x) = sin^4(\pi x)$.

Method	N	$L_\infty error$	$L_\infty order$	$L_1 error$	$L_1 order$
ENN	10	3.511e-1	—–	2.464e-1	—–
	20	7.458e-2	2.235	3.821e-2	2.689
	40	3.429e-2	1.121	1.454e-2	1.394
	80	7.545e-3	2.184	2.969e-3	2.292
	160	1.410e-3	2.420	4.424e-4	2.747
	320	3.351e-4	2.073	8.270e-5	2.419
WENN-5	10	2.823e-1	—–	1.801e-1	—–
	20	8.042e-2	1.812	4.159e-2	2.115
	40	5.836e-3	3.785	2.529e-3	4.040
	80	7.861e-4	2.892	2.627e-4	3.267
	160	3.260e-5	4.592	7.756e-6	5.082
	320	8.444e-7	5.271	2.055e-7	5.238
WENO-5	10	3.341e-1	—–	2.100e-1	—–
	20	1.070e-1	1.643	4.896e-2	2.101
	40	8.903e-3	3.587	3.635e-3	3.752
	80	1.699e-3	2.390	4.777e-4	2.928
	160	6.813e-5	4.640	1.482e-5	5.011
	320	1.760e-6	5.274	3.863e-7	5.261

Table 3: Comparasion of convergence and error on $u_t + uu_x = \frac{1}{Re}\frac{\partial^2 u}{\partial x^2}$, with $Re = 1000$.

Method	N	Convergent Times	$L_\infty error$	$L_1 error$
ENN	40	440	1.124e-2	5.673e-4
	80	843	2.219e-2	5.671e-4
WENN-5	40	436	1.073e-2	5.405e-4
	80	837	2.073e-2	5.302e-4
WENO-5	40	440	1.112e-2	5.623e-4
	80	842	2.211e-2	5.665e-4

(4) Shock-boundary layer interaction

The time-dependent two-dimensional Navier-Stokes equations in Cartesian coordinates have as follows:

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = \frac{\partial F_v}{\partial x} + \frac{\partial G_v}{\partial y} \tag{35}$$

where U is a vector of the conserved variables, F and G are the inviscid fluxes, Fv and Gv are the viscous fluxes.

$$\begin{aligned}
U &= (\rho, \rho u, \rho v, \rho e_t)^T \\
F &= (\rho u, \rho u^2 + p, \rho uv, (\rho e_t + p)u)^T \\
G &= (\rho v, \rho uv, \rho v^2 + p, (\rho e_t + p)v)^T \\
F_v &= (0, \tau_{xx}, \tau_{xy}, u\tau_{xx} + v\tau_{xy} + q_x)^T \\
G_v &= (0, \tau_{xy}, \tau_{yy}, u\tau_{xy} + v\tau_{yy} + q_y)^T \\
e_t &= e + \frac{1}{2}(u^2 + v^2)
\end{aligned}$$

The shear stresses have the form

$$\begin{aligned}
\tau_{xx} &= \frac{\mu}{R_{e\infty}} \cdot \frac{2}{3}(2u_x - v_y) \\
\tau_{xy} &= \frac{\mu}{R_{e\infty}} \cdot (u_y + v_x) \\
\tau_{yy} &= \frac{\mu}{R_{e\infty}} \cdot \frac{2}{3}(2v_y - u_x)
\end{aligned}$$

The heat fluxes are

$$\begin{aligned}
q_x &= \frac{\mu}{R_{e\infty}(\gamma - 1)M_\infty^2 P_r} \cdot T_x \\
q_y &= \frac{\mu}{R_{e\infty}(\gamma - 1)M_\infty^2 P_r} \cdot T_y
\end{aligned}$$

The equation of state is

$$T = \frac{\gamma M_\infty^2 P}{\rho}$$

The coefficient of viscosity μ is given by Sutherlandś formula:

$$\mu = T^{\frac{3}{2}} \frac{1 + C}{T + C}, C = \frac{110.4}{T_\infty}$$

where Pr is the Prandtl number, M_∞–the Mach number and $R_{e\infty}$–the Reynolds number. In this computation, we introduced a coordinate transformation as follows to satisfy a sufficient number of points within the viscous layer.

$$\begin{cases} \xi = x \\ \eta = 1 - \dfrac{ln(\dfrac{\beta + 1 - y_1}{\beta - 1 + y_1})}{ln(\dfrac{\beta + 1}{\beta - 1})}, \quad y_1 = y/h \end{cases} \tag{36}$$

In this paper, the explicit-implicit method [5] is adopted, i.e. the explicit method is applied in ξ-direction and the implicit method is applied in η-direction.

Computational parameters are

$$M_\infty = 2.0, Re_\infty = 2.96 \times 10^5, T_\infty = 293K$$
$$\beta = 1.002, h = 0.1215, \gamma = 1.4, Pr = 0.72$$

Following [4], computational domain is choosen to be $0 \le x \le 0.32, 0 \le y \le 0.1215$, the reference length $L = 0.16$, the impinging shock angle is $\theta = 32.585^o$.

Fig.3–fig.6 give the results by using ENN and WENN_5 schemes , fig.3 shows the distributions of pressure and fig.4 shows the distributions of the skin friction coefficient on the wall with 33×33 gridpoints, fig.5 and fig.6 with 65×65 gridpoints.

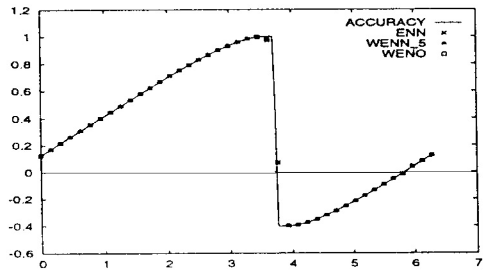

Fig. 1 N=40

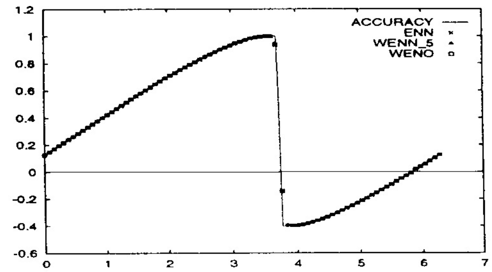

Fig. 2 N=80

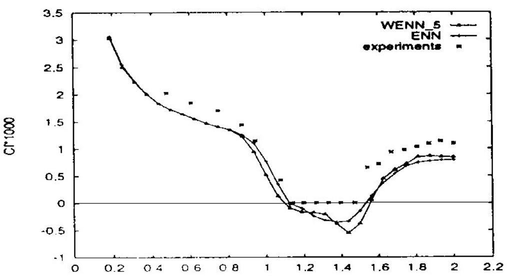

Fig. 3 Skin friction distributions on plate

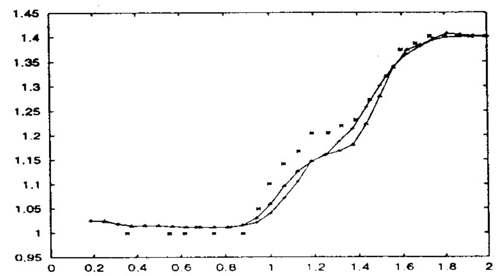

Fig. 4 Pressure distributions on plate

Comparision of the ENN and WENN_5 schemes for shock-boundary layer interaction(33X33)

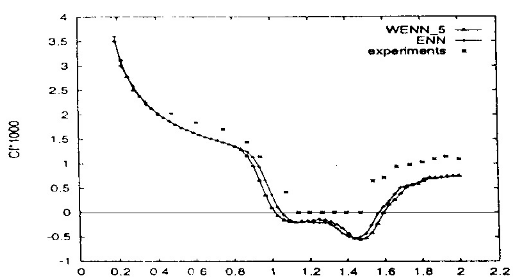

Fig. 5 Skin friction distributions on plate

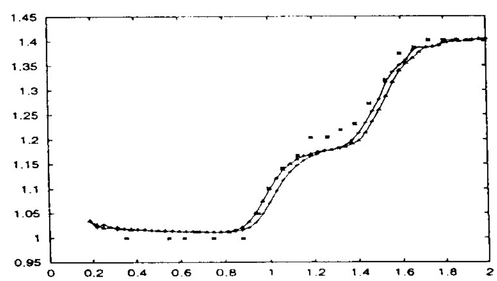

Fig. 6 Pressure distributions on plate

Comparision of the ENN and WENN_5 schemes for shock-boundary layer interaction(65X65)

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