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Mathematical Modelling of Malaria with Treatment

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Abstract. This paper proposes a Susceptible-Infective-Susceptible (SIS) model to study the malaria transmission with treatment by considering logistic growth of mosquito population. In this work, it is assumed that the treatment rate is proportional to the number of infectives below the capacity and is constant when the number of infectives is greater than the capacity. We find that the system exhibits backward bifurcation if the capacity is small and it gives bi-stable equilibria which makes the system more sensitive to the initial conditions. The existence and stability of the equilibria of the model are discussed in-detail and numerical simulations are presented to illustrate the numerical results.

AMS subject classifications: 92D30, 37N25

Key words: Malaria, treatment, simulation.

1 Introduction

The malaria is a mosquito-borne infectious disease of humans and other animals that is caused by protists (i.e., a type of microorganism) of the genus Plasmodium. Based upon the current understanding, there are four species of Plasmodium which are responsible for malaria in humans: P. Falciparum, P. Vivax, P. Ovale, and P. Malariae. Out of these, the majority of deaths are caused by P. Falciparum and P. Vivax. The remaining two, P. Ovale, and P. Malariae, cause a generally milder form of malaria that is rarely fatal. Furthermore, the zoonotic species P. Knowlesi, prevalent in Southeast Asia, causes malaria in macaques but can also cause severe infections in humans. Normally, malaria is significant in tropical and subtropical regions because of several reasons. For example, the heavy rainfall, warm temperatures, and stagnant waters provide habitats ideal for mosquito larvae. In this regard, the disease transmission can be reduced by preventing mosquito bites by distribution of mosquito nets and insect repellents, or with mosquito-control measures such as spraying insecticides and draining stagnant water.

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The P. Falciparum dominates in majority of malarial related deaths in Africa and South East Asia. It is responsible for nearly 80% of all malaria cases and nearly 90% of deaths, [1]. However in India roughly half of the cases of malaria are caused by the P. Falciparum, and half by the P. Vivax, [2]. In most of the tropical countries including India, the emergence of malaria has taken place and it has become endemic in the North-Eastern part of India, where this disease is spread by a lethal parasite called the Plasmodium Falciparum. Though, there are several experimental studies related to the surveys of malaria in different regions (see [3–10]), but the dynamics of malaria is very complex and there is a strong need to understand the transmission dynamics of malaria and the environmental factors which influence it.

Mathematical modeling is very helpful in understanding the dynamics of any infectious diseases and malaria is one of the diseases which is studied efficiently using modeling approaches. Sir Ronald Ross was the first to formulate a mathematical model for the P. Falciparum malaria in India by involving both, the human and mosquito population, [11]. This model was very simple and later it was modified by several researchers, (see [12–20]). Mathematical modeling of control of malaria by considering different aspects of controls has been discussed in [21–24].

Treatment of infective is an important parameter to control the spread of the disease related to malaria. In the classical epidemic models, the treatment rate of infectives is assumed to proportional to the number of infectives but this fact is applicable to the developing countries which have limited resources. As in case of limited resources, it is not possible to provide treatment to all infectives if the size of infective class is very large. Hence here we apply following treatment rate function which is described in [25]:

$$T(I)=\begin{cases} rI, & \text{if } 0\le I\le I_0, \\ k, & \text{if } I>I_0. \end{cases} \tag{1.1}$$

Where I denotes infective class and $k=rI_0$. This means that the treatment rate is proportional to the number of the infectives when the capacity of treatment is not reached, and otherwise takes the maximal capacity. This describes the situation where patients have to be hospitalized: the number of hospital beds is limited. This is true also for the case where medicines are not sufficient.

In this paper, therefore, an SIS non-linear mathematical model is proposed and analyzed by incorporating treatment. In most of the existing malaria models, the population demography for mosquitoes has been assumed as of constant recruitment and death type. In the present work, we assume that the density of the mosquito population follows a generalized logistic model such that its growth rate decreases but its death rate increases as population density increases towards its carrying capacity with respect to the environment. This assumption is more realistic and reasonable as the mosquito population density is high in the regions that are conducive to its growth such as rivers and ponds as well as man-made habitats e.g., water storage tank, rice fields, barrels, irrigation channels, ditches, field wells etc. We analyze the model using the stability theory of