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A New Boundary Condition for the Three-Dimensional MHD Equation and the Vanishing Viscosity Limit

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Abstract. In this paper, we consider the viscous incompressible magnetohydrodynamic (MHD) system with a new boundary condition for a general smooth domain in $\mathbb{R}^3$. We obtain the well-posedness of the system and the vanishing viscosity limit result.

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1 Introduction

Let $\Omega\subset\mathbb{R}^3$ be a bounded smooth domain, and satisfy the topology condition $\mathrm{H}_1(\Omega,R)=0$. We study the following MHD system in Ω:

$$\begin{cases}\partial_t u-\varepsilon\Delta u+u\cdot\nabla u-b\cdot\nabla b+\nabla p=0,\\ \partial_t b-\varepsilon\Delta b+u\cdot\nabla b-b\cdot\nabla u=0,\\ \nabla\cdot u=0,\ \nabla\cdot b=0,\\ u(0,x)=u_0(x),\ b(0,x)=b_0(x),\end{cases}\tag{1.1}$$

with the boundary condition on $\partial\Omega$:

$$\begin{cases}u\cdot n=0,\quad w\cdot n=0,\quad n\times(\Delta u)=0,\\ b\cdot n=0,\quad j\cdot n=0,\quad n\times(\Delta b)=0,\end{cases}\tag{1.2}$$

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where u,b,p denote the velocity, magnetic field and pressure, respectively, ε represents the viscosity and magnetic diffusion coefficient. And $w:=\nabla\times u$, $j:=\nabla\times b$. This boundary condition is motivated by the generalized Navier-slip boundary condition involving the vorticity.

The corresponding ideal MHD system is

$$\begin{cases} \partial_t u+u\cdot\nabla u-b\cdot\nabla b+\nabla p=0,\\ \partial_t b+u\cdot\nabla b-b\cdot\nabla u=0,\\ \nabla\cdot u=0,\ \nabla\cdot b=0,\\ u(0,x)=u_0(x),\quad b(0,x)=b_0(x), \end{cases} \tag{1.3}$$

with the slip boundary condition:

$$u\cdot n=0,\quad b\cdot n=0. \tag{1.4}$$

We want to study the well-posedness of solutions to (1.1)-(1.2) and whether they converge to those of ideal MHD system (1.3)-(1.4) as $\varepsilon\to 0$.

For the common viscous incompressible MHD system(i.e.the viscosity coefficient and magnetic diffusion coefficient may not be equal), there are a lot of literatures on various topics concerning solvability, regularity in the whole space or with slip boundary conditions, see [1–9], and therein references. For the vanishing viscosity limit problem, we can find quite a large literature for Navier-Stokes equations, such as [10–18]. However, there are not so many results for MHD equations. In [8], the authors consider the system in a cubic domain with the following slip boundary conditions:

$$\begin{cases} u\cdot n=0,\quad n\times w=0,\\ b\cdot n=0,\quad n\times j=0. \end{cases}$$

When Ω is a bounded smooth domain, Guo and Wang in [19] investigate the vanishing viscosity limit problem of 3D MHD with the generalized Navier slip boundary condition:

$$\begin{cases} u\cdot n=0,\quad n\times w=[\beta u]_\tau,\\ b\cdot n=0,\quad n\times j=[\beta b]_\tau, \end{cases}$$

where β is a given smooth symmetric tensor on the boundary. When the Dirichlet boundary condition is given,there exists boundary layer which is still a challenge problem in the fluid dynamics. Xie, et al. [20] study the MHD equations with the non-zero Dirichlet boundary conditions, and obtain that the solution of MHD equations converges to that of the ideal MHD equations as viscosity and magnetic diffusion coefficient tends to zero.

In order to avert the appearance of the boundary layer and want to obtain a better result, we propose the new boundary condition (1.2) for Eqs. (1.1) as in [21].

We organize the paper as follows. In Section 2, we give the main results. In Section 3, we verify the local well-posedness of system (1.1)-(1.2). In Section 4, we obtain the convergence rates of the solutions for (1.1)-(1.2) to the corresponding ideal MHD equations (1.3)-(1.4).