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AN ALGORITHM FOR FINDING GLOBAL MINIMUM OF NONLINEAR INTEGER PROGRAMMING*

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Abstract

A filled function is proposed by R.Ge[2] for finding a global minimizer of a function of several continuous variables. In [4], an approach for finding a global integer minimizer of nonlinear function using the above filled function is given. Meanwhile a major obstacle is met: if $\rho > 0$ is small, and $\|x_I - \overset{*}{x}_I\|$ is large, where x_I - an integer point, $\overset{*}{x}_I$ - a current local integer minimizer, then the value of the filled function almost equals zero. Thus it is difficult to recognize the size of the value of the filled function and can not to find the global integer minimizer of nonlinear function. In this paper, two new filled functions are proposed for finding global integer minimizer of nonlinear function, the new filled function improves some properties of the filled function proposed by R. Ge [2].

Some numerical results are given, which indicate the new filled function (4.1) to find global integer minimizer of nonlinear function is efficient.

Key words: Local integer minimizer, Global integer minimizer, Filled function.

1. Introduction

It is well known, the problem for linear integer programming is an NP hard problem [3]. Certainly, nonlinear integer programming is more difficult than linear integer programming. There is no any algorithm for finding global minimizer of integer programming with quadratic constraints and linear objective function [1]. Therefore, generally assume that the domain of function is bounded when nonlinear integer programming is concerned. Even for this case, there also is lack of the algorithms to solve it.

In [4], we proposed an approach for finding global minimum of nonlinear integer programming (P) using the filled function method [2].

$$\text{(P)} \qquad \min_{x_I \in X_I} f(x_I)$$

where $X \subset R^n$-a bounded box, X_I-an integer point set in X, R_I^n- all integer points in R^n.

Redefined function $f(x_I)$,

$$f(x_I) = \begin{cases} f(x_I), & x_I \in X_I, \\ +\infty, & x_I \in R_I^n \setminus X_I. \end{cases}$$

Thus problem (P) is equivalent to the following problem (P)_I

$$\text{(P)}_I \qquad \min_{x_I \in R_I^n} f(x_I) \tag{1.1}$$

The filled function is as follows

$$p(x_I, \overset{*}{x}{}_I^1, r, \rho) = \begin{cases} \dfrac{1}{r + f(x_I)} \exp\Big(-\dfrac{\|x_I - \overset{*}{x}{}_I^1\|^2}{\rho}\Big), & x_I \in X_I, \\ +\infty & x_I \in R_I^n \setminus X_I \end{cases} \tag{1.2}$$

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Where $r + f(\overset{*1}{x}_I) > 0, \rho > 0, \overset{*1}{x}_I$ is a discrete local minimizer of $(P)_I$. We say that an integer point $\overset{*1}{x}_I$ is a (strictly) discrete local minimizer of $f(x_I)$, if $f(\overset{*1}{x}_I) \le (<) f(\overset{*1}{x}_I \pm e_i), i = 1, \cdots, n$. Here $e_i = (0, \cdots, 0, 1, 0, \cdots, 0)$.

In fact, the filled function $p(x_I, \overset{*1}{x}_I, r, \rho)$ of (1.1) is

$$p(x_I, \overset{*1}{x}_I, r, \rho) = \begin{cases} \dfrac{1}{r + f(x_I)} \exp\Big(- \dfrac{\|x_I - \overset{*1}{x}_I\|^2}{\rho}\Big), & x_I \in X_I, \\ 0 & x_I \in R_I^n \setminus X_I \end{cases} \tag{1.3}$$

But from the view point of computition, it is unnecessary to search in $R_I^n \setminus X_I$, thus we use (1.2) as the filled function.

For the filled function (1.2), in theory, we prove that if we suitably choose $r, \rho > 0$, then by the procedure of finding local minimum of (1.2) we can find a point $\overset{*1}{\bar{x}}_I$ satisfying $f(\overset{*1}{\bar{x}}_I) \le f(\overset{*1}{x}_I)$, thus by using $\overset{*1}{\bar{x}}_I$ as an initial point, we can find another discrete local minimizer $\overset{*2}{x}_I$ of (1.1) satisfying $f(\overset{*2}{x}_I) < f(\overset{*1}{x}_I)$. Using $\overset{*2}{x}_I$, we can construct a new filled function (1.2) $p(x_I, \overset{*2}{x}_I, r, \rho)$. Repeating the above procedure, finally we can find the global minimizer x_I^* of (1.1).

When we test this approach on some functions, one major disadvantage appears: If $\rho > 0$ is small, and $\|x_I - \overset{*1}{x}_I\|$ is large, then the value of $\exp\big(- \frac{\|x_I - \overset{*1}{x}_I\|^2}{\rho}\big)$ almost equals zero, thus it is difficult to recognize the size of $\exp\big(- \frac{\|x_I - \overset{*1}{x}_I\|^2}{\rho}\big)$, and hence the discrete local minimum of $p(x_I, \overset{*1}{x}_I, r, \rho)$.

In order to overcome it, we will propose the following filled function with one parameter:

$$p(x, x^*, \rho) = f(x^*) - f(x) - \rho\|x - x^*\|^2 \tag{1.4}$$

2. Some Definitions and an Algorithm of Finding the Local Discrete Minimum

Definition 2.1. *For any $x_I \in R_I^n$, the neighbour of x_I is defined by $N(x_I) = \{x_I, x_I \pm e_i, i = 1, \cdots, n\}$, where e_i denotes the unit vector whose i-th component equals 1, and other components equal 0.*

Definition 2.2. *It is said that the integer point $x_I^0 \in R_I^n$ is a (strictly) discrete local minimizer of $f(x_I)$, if $f(x_I^0) \le (<) f(x_I)$ for all $x_I \in N(x_I^0)$.*

Definition 2.3. *It is said that the integer point $x_I^0 \in R_I^n$ is a (strictly) discrete global minimizer of $f(x_I)$, if $f(x_I^0) \le (<) f(x_I)$ for all $x_I \in R_I^n$.*

Obviously, a discrete global minimum of $f(x_I)$ must also be a discrete local minimum of $f(x_I)$.

Similarly, a discrete local maximum and discrete global maximum of $f(x_I)$ can be defined.

Let $D = \{e_i, -e_i, i = 1, \cdots, n\}$.

Definition 2.4. *It is said that $d_0 \in D$ is a descent direction of $f(x_I)$ at $x_I^0 \in R_I^n$, if $f(x_I^0 + d_0) < f(x_I^0)$.*

A discrete local minimum of $(P)_I$ can be found by the following algorithm.

Algorithm 2.1.

Step 1. Select any initial point $x_I^0 \in X_I \subseteq R_I^n$.

Step 2. If x_I^0 is a discrete local minimizer of $(P)_I$, then stop. Otherwise, a descent direction d_0 of $f(x_I)$ at x_I^0 can be found.

Step 3. Let $x_I^0 := x_I^0 + d_0$, go to step 2.