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An Efficient Monte Carlo-Transformed Field Expansion Method for Electromagnetic Wave Scattering by Random Rough Surfaces

Xiaobing Feng[1], Junshan Lin[2,*] and David P. Nicholls[3]

[1] *Department of Mathematics, The University of Tennessee, Knoxville, TN 37996, USA.*
[2] *Department of Mathematics and Statistics, Auburn University, Auburn, AL 36849, USA.*
[3] *Department of Mathematics, Statistics, and Computer Science, University of Illinois at Chicago, Chicago, IL 60607, USA.*

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Abstract. This paper develops an efficient and accurate numerical method for the computation of electromagnetic waves scattered by random rough surfaces. The method is based upon a combination of the Transformed Field Expansion method, which represents the solution as a provably convergent power series, and the Monte Carlo technique for sampling the probability space. The compelling aspect of the proposed method is that, at each perturbation order and every sample, the governing Transformed Field Expansion equations share the same deterministic Helmholtz operator on a deterministic domain. Thus, an LU factorization of the matrix discretization of this single operator can be employed repeatedly for all orders and every sample. Consequently, the computational complexity of the whole algorithm is significantly reduced as a result. Numerical examples are described which demonstrate the accuracy of the algorithm.

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1 Introduction

The scattering of electromagnetic waves from random rough surfaces has long been a subject of interest due to its significant applications in remote sensing, oceanography,

*Corresponding author. *Email addresses:* `xfeng@utk.edu` (X. Feng), `jzl0097@auburn.edu` (J. Lin), `davidn@uic.edu` (D. P. Nicholls)

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surface plasmonics, solar cells, etc. [13, 17, 19, 23, 28, 30]. If the deviation of the surface shape from trivial (flat) is small then asymptotic techniques such as the perturbation or Kirchhoff theories can be applied to derive the analytical solutions with reasonable accuracy [5, 26, 29]. However, when the surface is a large and/or rough deviation from trivial then analytical methods are insufficient to deliver desired error tolerances and one must resort to numerical simulation of the electromagnetic wave propagation. The simplest, and most natural, numerical approach is to use the Monte Carlo (MC) method where a set of numerical solutions are obtained for independent identically distributed (*i.i.d.*) sample surface profiles which are subsequently utilized to calculate the statistics of the scattered waves [14, 18, 25]. An alternative approach is to transform the scattering problem on a random domain into a stochastic problem on a deterministic domain, which is then solved with either Monte Carlo simulations or stochastic Galerkin methods (c.f. [3, 4, 10, 34–36]). In typical simulations both of these methods become computationally intractable when a large number of degrees of freedom is required for the spatial discretization: The MC method requires solution of the full scattering problem many times with different sampling surfaces, while the stochastic Galerkin method (which is based upon the representation of the random solution by either the Karhunen-Loève or Wiener Chaos expansion) usually leads to a high-dimensional, deterministic equations that are too expensive to solve. We also refer to [7, 8] for other efficient numerical methods to solve elliptic equations with random domains.

In this paper, we propose an efficient Monte Carlo-Transformed Field Expansion (MC-TFE) method for the simulation of electromagnetic wave scattering by random rough surfaces. More precisely, a High-Order Perturbation of Surfaces (HOPS) Taylor expansion [24] is employed to represent the solution for every realization in a Monte Carlo method which is applied to sample the relevant probability space. A change of variables, which flattens the problem domain, generates a Helmholtz equation at every perturbation order with deterministic coefficients and random sources, which is posed on a deterministic domain. We apply a High Order Spectral Legendre-Galerkin method [31] to discretize the deterministic Helmholtz operator. In addition, to accelerate the algorithm, we take advantage of the fact that, at every perturbation order and every random sample, the same deterministic differential operator must be approximated. By performing an LU decomposition of the discretization matrix of the operator, all samples at every order can be obtained in an efficient way by simple forward and backward substitutions, thereby significantly reducing the computational cost. Similar expansion techniques and acceleration schemes have also been used by the authors for the modeling of waves in random media. The interested reader is referred to [11, 12] for complete details.

The rest of the paper is organized as follows. We introduce the governing equations for the scattering problem in Section 2. The computational modeling of random surfaces is briefly discussed in Section 3, and the MC-TFE method is presented and discussed in Section 4. Several numerical experiments are provided in Section 5 to demonstrate the accuracy and reliability of the method.