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Backward Error Analysis for an Eigenproblem Involving Two Classes of Matrices

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Abstract. We consider backward errors for an eigenproblem of a class of symmetric generalised centrosymmetric matrices and skew-symmetric generalised skew-centrosymmetric matrices, which are extensions of symmetric centrosymmetric and skew-symmetric skew-centrosymmetric matrices. Explicit formulae are presented for the computable backward errors for approximate eigenpairs of these two kinds of structured matrices. Numerical examples illustrate our results.

AMS subject classifications: 65F10, 65F15

Key words: Backward error, symmetric generalised centrosymmetric matrix, skew-symmetric generalised skew-centrosymmetric matrix.

1. Introduction

It is well-known that backward errors are very important for assessing the stability and quality of numerical algorithms. In this article, we consider backward errors for an eigenproblem of a special class of symmetric generalised centrosymmetric matrices and skew-symmetric generalised skew-centrosymmetric matrices, with practical applications. For example, a small perturbation method and backward errors for an eigenproblem were key techniques for a nonlinear component level model, and a state variables linear model of a turbofan engine — cf. [16–18].

Let $\mathscr{C}$ and $\mathscr{C}^{m\times n}$ denote the set of complex numbers and $m \times n$ complex matrices, respectively. (We will abbreviate $\mathscr{C}^{m\times 1}$ as $\mathscr{C}^{m}$.) The conjugate, transpose, conjugate transpose and Moore-Penrose generalised inverse of a matrix A are denoted by $\bar{A}$, A^T, A^* and A^+, respectively. The identity matrix of order n is denoted by I_n; the matrix norm adopted is

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the Frobenius norm defined by $\|A\|_F = \sqrt{tr(A^*A)}$; and P_A and $P_A^\perp$ denote the orthogonal projection onto $\mathscr{R}(A)$ and the projection complementary to P_A, respectively. We also write $\mathscr{O}\mathscr{C}^{m\times m} = \{A \in \mathscr{C}^{m\times m} | A^T A = AA^T = I_m\}$.

Definition 1.1 (cf. Ref. [1]). Let $A, B \in \mathscr{C}^{k\times k}$, $\mu, \nu \in \mathscr{C}^k$, $\beta \in C$ and assume $P \in \mathscr{C}^{k\times k}$ is nonsingular. Then the block matrices

$$\mathscr{A}_{2k} = \begin{pmatrix} A & BP \\ P^{-1}B & P^{-1}AP \end{pmatrix} \qquad (k \geq 1),$$

$$\mathscr{A}_{2k+1} = \begin{pmatrix} A & \mu & BP \\ \nu^T & \beta & \nu^T P \\ P^{-1}B & P^{-1}\mu & P^{-1}AP \end{pmatrix} \ (k \geq 0),$$

are called $2k$, $2k+1$ step generalised centrosymmetric matrices and denoted by $\mathscr{G}\mathscr{C}^{2k\times 2k}$ and $\mathscr{G}\mathscr{C}^{(2k+1)\times(2k+1)}$, respectively. Similarly,

$$\mathscr{B}_{2k} = \begin{pmatrix} A & BP \\ -P^{-1}B & -P^{-1}AP \end{pmatrix} \qquad (k \geq 1),$$

$$\mathscr{B}_{2k+1} = \begin{pmatrix} A & \mu & BP \\ -\nu^T & \beta & \nu^T P \\ -P^{-1}B & -P^{-1}\mu & -P^{-1}AP \end{pmatrix} \ (k \geq 0),$$

are called $2k$, $2k+1$ step generalised skew-centrosymmetric matrices and denoted by $\mathscr{G}\tilde{\mathscr{C}}^{2k\times 2k}$ and $\mathscr{G}\tilde{\mathscr{C}}^{(2k+1)\times(2k+1)}$, respectively.

Definition 1.2 (cf. Ref. [6]). We define $\mathscr{S}\mathscr{G}\mathscr{C}^{m\times m} = \{A \in \mathscr{G}\mathscr{C}^{m\times m} | A = A^T\}$ and $\tilde{\mathscr{S}}\mathscr{G}\tilde{\mathscr{C}}^{m\times m} = \{A \in \mathscr{G}\tilde{\mathscr{C}}^{m\times m} | A = -A^T\}$ — i.e. as the sets of symmetric generalised centrosymmetric matrices and skew-symmetric generalised skew-centrosymmetric matrices, respectively.

In Definition 1.1, P is restricted to be orthogonal; and the corresponding classes of symmetric generalised centrosymmetric matrices and skew-symmetric generalised skew-centrosymmetric matrices are denoted by $\mathscr{K}_1$ and $\mathscr{K}_2$, respectively. These classes of symmetric generalised centrosymmetric matrices and skew-symmetric generalised skew-centrosymmetric matrices have practical applications in aerostatics, information theory, linear system theory, and linear estimate theory [1–6]. We can obtain the block forms of $\mathscr{K}_1$ and $\mathscr{K}_2$ as follows (for a proof see Lemmas 2.3 and 2.6 below):
for $2k$ ($k \geq 1$),

$$\mathscr{K}_1 = \left\{ \begin{pmatrix} A_1 & BP_0 \\ P_0^{-1}B & P_0^{-1}A_1P_0 \end{pmatrix} \right\}, \quad \mathscr{K}_2 = \left\{ \begin{pmatrix} A_2 & BP_0 \\ -P_0^{-1}B & -P_0^{-1}A_2P_0 \end{pmatrix} \right\};$$

for $2k+1$ ($k \geq 0$),

$$\mathscr{K}_1 = \left\{ \begin{pmatrix} A_1 & \mu & BP_0 \\ \mu^T & \beta & \mu^T P_0 \\ P_0^{-1}B & P_0^{-1}\mu & P_0^{-1}A_1P_0 \end{pmatrix} \right\},$$