## Thermodynamical Bending of FGM Sandwich Plates Resting on Pasternak's Elastic Foundations

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**Abstract.** The analysis of thermoelastic deformations of a simply supported functionally graded material (FGM) sandwich plates subjected to a time harmonic sinusoidal temperature field on the top surface and varying through-the-thickness is illustrated in this paper. The FGM sandwich plates are assumed to be made of three layers and resting on Pasternak's elastic foundations. The volume fractions of the constituents of the upper and lower layers and, hence, the effective material properties of them are assumed to vary in the thickness direction only whereas the core layer is still homogeneous. When in-plane sinusoidal variations of the displacements and the temperature that identically satisfy the boundary conditions at the edges, the governing equations of motion are solved analytically by using various shear deformation theories as well as the classical one. The influences of the time parameter, power law index, temperature exponent, top-to-bottom surface temperature ratio, side-to-thickness ratio and the foundation parameters on the dynamic bending are investigated.

## AMS subject classifications: 74XX

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## 1 Introduction

Sandwich structures have high structural efficiency because of their excellent properties such as high ratio of strength-to-weight, good energy and sound absorption capability, and often low production cost. They are mainly used in aerospace, marine and aircraft

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industry as thin-walled structures but at present the application of these structures has been extended to automobile, petrochemical and other industries. Structural sandwich is basically fabricated from three layers. The two face sheets adhesively bonded to the core. The traditional sandwich structures have some drawbacks. The sudden change in material properties across the interfaces among different materials can result in large interlaminar stresses. To overcome these disadvantages, FGMs composed of two or more phases with different material properties and continuously varying composition distribution have been used as a core layer or face layers. Such materials were introduced to take advantage of the desired material properties of each constituent material without interface problems. Owing to these reasons, a number of research works about the sandwich structures with FGM face layers have been established, in particular to study their elastic [1] and thermo-elastic behavior [2–4]. To investigate the effect of FGM core on performance of sandwich plates, Anderson [5] and Kashtalyan and Menshykova [6] have developed the 3-D elasticity solution for sandwich composites with a FG core subjected to transverse loading.

FGMs are widely used in many engineering applications, for example aerospace, automotive and biomedical applications. Thus, many works on FGM structures have been studied in literature. For example, Reddy [7] has analyzed the static behavior of FG rectangular plates based on his third-order shear deformation plate theory. Reddy and Chen [8] have presented a three-dimensional model for a FG plate subjected to mechanical and thermal loads, both applied at the top of the plate. Vel and Batra [9] have proposed a three-dimensional solution for transient thermal stresses in FG rectangular plates. Also, several investigations on the behavior of the FGM plates, disks and cylinders have been explained in Zenkour [10–14], Zenkour et al. [15] and Zenkour and Sobhy [16,17].

Structures resting on elastic foundations is often encountered in the analysis of the foundations of buildings, highway and railroad structures, and of geotechnical structures [18]. The simplest and most frequently employed elastic foundation model is that of Winkler [19], which is generally referred to as a one-parametric model. The transverse deformation characteristics of the elastic foundation are defined by means of continuous and closely spaced linear springs providing resistance in direct proportion to the deflection of the plate. The deficiency of Winkler's formulation is the behavioral inconsistency due to the discontinuity of displacements on the boundary of the uniformly loaded surface area [20]. To add the influence of shear effect of the foundation besides the vertical springs, Pasternak [21] had introduced a shear layer such that it is an incompressible vertical element and deforms only by transverse shear force. Many formulations of the equations of dynamic and static equilibrium of structures on Winkler's elastic foundation [22–24] or Pasternak's ones [25–29] are to be encountered in the literature.

Thermal effects could be important when a mechanical system has undergo high or low temperature gradients. Thus, the effect of thermal loading on the displacement and stress fields for FGM plates and shells has been studied by a number of authors. Praveen and Reddy [30] have employed the finite element method to illustrate the response of