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Comparison of Simulations of Convective Flows

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Abstract. We show that a single particle distribution for the "energy-conserving" D2Q13 lattice Boltzmann scheme can simulate coupled effects involving advection and diffusion of velocity and temperature. We consider various test cases: non-linear waves with periodic boundary conditions, a test case with buoyancy, propagation of transverse waves, Couette and Poiseuille flows. We test various boundary conditions and propose to mix bounce-back and anti-bounce-back numerical boundary conditions to take into account velocity and temperature Dirichlet conditions. We present also first results for the de Vahl Davis heated cavity. Our results are compared with the coupled D2Q9-D2Q5 lattice Boltzmann approach for the Boussinesq system and with an elementary finite differences solver for the compressible Navier-Stokes equations. Our main experimental result is the loss of symmetry in the de Vahl Davis cavity computed with the single D2Q13 lattice Boltzmann model without the Boussinesq hypothesis. This result is confirmed by a direct Navier Stokes simulation with finite differences.

AMS subject classifications: 6505, 76N15, 80A20, 82C20

Key words: Bounce-back, natural convection, adiabatic wall, de Vahl Davis.

1 Introduction

Lattice Boltzmann schemes have proven their efficiency for the computation of quasi-incompressible flows. We refer *e.g.* to [2, 9, 11] among others. In these cases, the physical conservations of mass and momentum are implemented in the framework of lattice Boltzmann schemes. When compressible effects are taken into account, it is necessary to

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add the conservation of energy. A classical approach is to begin with weakly compressible effects that can be modelled with the so-called Boussinesq approximation. In this case, the incompressibility condition remains a good approximation and coupled effects between conservations of momentum and energy are taken into account with a precise thermodynamical analysis. We refer to Landau [14] or Batchelor [1] for the derivation of the Boussinesq approximation. The implementation of the Boussinesq approximation is possible with the lattice Boltzmann approach with the introduction of two particle distributions. This idea has been also proposed in the context of finite volumes by the team of Perthame [10], and with lattice Boltzmann schemes by Eggels and Somers [5], Mezrhab *et al.* [16] and Wang *et al.* [21] among others.

In this contribution, we study a direct approximation of the compressible Navier Stokes equations with an "energy-conserving" lattice Boltzmann scheme using a single particle distribution. A first tentative study [12] has shown that for a critical value of the Prandlt number, the thermal wave and the viscous one merge together, the physics is badly represented and an instability occurs in general. In consequence, no satisfying compressible flows have been obtained with this direct numerical modelling. In a second tentative [13], we have analyzed with great details several lattice Boltzmann schemes with four conservation laws in two space dimensions. With an adequate fitting of the parameters of the scheme, it is possible to enlarge the zone in the spectral space where the thermal and viscous waves remain decoupled. Moreover, these parameters guarantee also the isotropy of the acoustic waves. Our objective is to enlarge the domain of validity of our previous study: incorporate the treatment of boundary conditions with rigid walls with a given temperature or adiabatic boundaries, study several couplings between velocity and temperature for elementary Couette and Poiseuille flows, study the possibility of Dirichlet and Neumann boundary conditions. Finally, our objective is the simulation of the de Vahl Davis test case [20] described in Fig. 1.

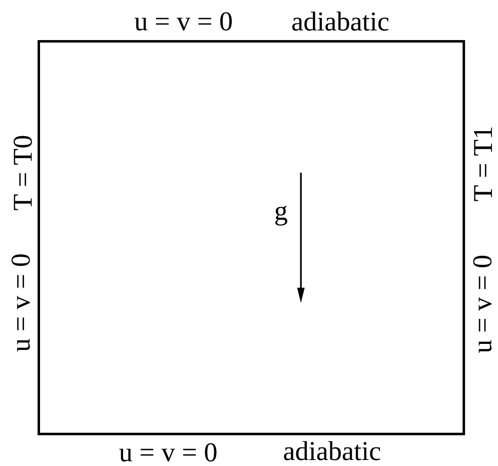

Figure 1: De Vahl Davis test case for natural convection.