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Numerical Simulation of the Motion of Inextensible Capsules in Shear Flow Under the Effect of the Natural State

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Abstract. In this paper, a computational model for the natural state of an inextensible capsule has been successfully combined with a spring model of the capsule membrane to simulate the motion of the capsule in two-dimensional shear flow. Besides the viscosity ratio of the internal fluid and external fluid of the capsule, the natural state also plays a role for having the transition between two well known motions, tumbling and tank-treading (TT) with the long axis oscillates about a fixed inclination angle (a swinging mode), when varying the shear rate. Between tumbling and tank-treading, the intermittent behavior has been obtained for the capsule with a biconcave rest shape. The estimated critical value of the swelling ratio for having the intermittent transition behavior is less than 0.7, i.e., the capsules with rest shape closer to a full disk do not have the intermittent behavior in shear flow. The intermittent dynamics of the capsule in the transition region is a mixture of tumbling and TT with a swinging mode. Just like the motion of TT with a swing mode, which can be viewed as a tank-treading with an incomplete tumbling, the membrane tank-treads backward and forward within a small range during the tumbling motion.

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Key words: Natural state, tumbling, tank-treading with a swinging mode, intermittent region, capsule, shear flow.

1 Introduction

The dynamical behavior of deformable entities such as lipid vesicles, capsules, and red blood cells (RBCs) in flows has received increasing attention experimentally, theoretically, and numerically in recent years. Lipid vesicles [1–10], non-spherical capsules [11–15],

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and red blood cells [16–24] show phenomenologically similar behavior in shear flows, which are (i) an unsteady tumbling motion, (ii) a tank-treading rotation with a stationary shape and a fixed inclination angle with respect to the flow direction, and (iii) while tank-treading (TT), the long axis oscillates about a fixed inclination angle which is called swinging mode in [9] (but it is still called the tank-treading in [12]). The aforementioned motions depend on the shear rate, viscosity ratio between internal fluid and external fluid, membrane viscosity and other parameters. Noguchi and Gompper [7–9] have studied the dynamics of vesicles in simple shear flow using mesoscale simulations of dynamically triangulated surfaces. Vesicles are found to have a vacillating breathing mode when transiting from steady tank-treading to unsteady tumbling with increasing membrane viscosity or the shear rate. At the vacillating breathing mode, the main axis makes oscillation around the flow direction (around zero degree angle, but with large amplitude). In [9], they have developed a model based on the classical model of Keller and Skalak [11] (KS model) for the membrane to theoretically explain the vesicle motion. Using such model, they obtained the swinging mode and studied the dependency of transition between tumbling and TT with a swinging mode on the shear rate, the viscosity ratio of the membrane and the internal fluid, and the reduced volume [9]. Abkarian *et al.* [19] observed the intermediate motions at the transition from swinging to tumbling (resp., tumbling to swinging) by reducing (resp., increasing) the flow shear rate. A simplified model with a fixed elliptical shape for cell membrane has been studied to support their observations. In [12], Skotheim and Secomb introduced an elastic energy term based on the phase angle of the tank-treading rotation to the KS model. They observed tumbling, tank-treading (with a swinging mode), and the intermittent behavior at the transition between tumbling and tank-treading and analyzed the influence of the viscosity ratio, membrane elasticity, shape, and the shear rate on the motion of a capsule of either prolate or oblate shape. The both models considered in [12, 19] take into account the membrane elastic energy. Tsubota *et al.* [22] used a spring model fully coupled with fluid flow to study cell motion dependency on the natural state of membrane through the bending energy term in two-dimensional shear flow. When being at uniform natural state (i.e., all reference angles as defined in Section 2.1 are set to be the same), the cell appears to tank-tread with an inclination angle unchanged and independent of the preset value of reference angles. But when a biconcave resting shape is assumed as the natural state (called non-uniform state), the cell is observed to perform tumbling and tank-treading with a swinging mode. Tsubota *et al.* [22] believed that the intermittency between tumbling and TT with a swinging mode would occur in very narrow range of parameter space; but they did not obtain such intermittent region because, as discussed later in this paper, the swelling ratios of their cells are not small enough. The elastic energy term introduced by Skotheim and Secomb in [12] has a similar characteristics like the bending energy defined with a non-uniform state used by Tsubota *et al.* in [22] since both have their preferred rest states, which are preferred phase angles at zero and 180 degrees and a biconcave resting shape for Skotheim and Secomb's model and Tsubota *et al.*'s model, respectively. In [14], Vlahovska *et al.* used perturbation approach to study the