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Deformation of a Sheared Magnetic Droplet in a Viscous Fluid

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Abstract. A fully three-dimensional numerical study of the dynamics and field-induced deformation of a sheared, superparamagnetic ferrofluid droplet immersed in a Newtonian viscous fluid is presented. The system is a three-dimensional, periodic channel with top and bottom walls displaced to produce a constant shear rate and with an external, uniform magnetic field perpendicular to the walls. The model consists of the incompressible Navier-Stokes equations with the extra magnetic stress coupled to the static Maxwell's equations. The coupled system is solved with unprecedented resolution and accuracy using a fully adaptive, Immersed Boundary Method. For small droplet distortions, the numerical results are compared and validated with an asymptotic theory. For moderate and strong applied fields, relative to surface tension, and weak flows a large field-induced droplet deformation is observed. Moreover, it is found that the droplet distortion in the vorticity direction can be of the same order as that occurring in the shear plane. This study highlights the importance of the three-dimensional character of a problem of significant relevance to applications, where a dispersed magnetic phase is employed to control the rheology of the system.

AMS subject classifications: 76T20

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1 Introduction

Magnetic fluids, also known as *ferrofluids*, are synthetic suspensions composed of solid magnetic particles coated with a surfactant and suspended in a liquid carrier. In general, particles are of magnetite with a diameter on the order of 3 *nm* to 15 *nm* (nanoparticles)

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and the liquid carrier is typically oil or water (a Newtonian fluid). Thermal agitation keeps particles suspended due to Brownian motion while a surfactant (or electrically charged particles) prevents them from clustering together [1–3].

An applied magnetic field, even of moderate intensity, can dramatically change the rheology of a ferrofluid. This is significant for a number of applications in which the magnetic component phase is used for controlling the system. In particular, two-phase systems consisting of ferrofluid droplets immersed in a Newtonian carrier have a great potential in important technological applications such as in the design of new, functional materials [4] and in the delivery of drugs [5, p. 233] [6–8].

Several numerical investigations have been devoted to the deformation of ferrofluid droplets under different flow and field conditions. Most studies have been either two-dimensional or axi-symmetric [9–15]. Interface instabilities of a ferrofluid droplet under the influence of an imposed magnetic field have also received considerable attention [16–18].

In this work, we focus on the field-induced distortion of a ferrofluid droplet in shear flow. We are inspired by the work of [14] on such deformation in quiescent flow. To this end, we perform fully three-dimensional numerical simulations employing an adaptive Immersed Boundary Method. This is coupled to an efficient, adaptive finite difference solver for the static Maxwell's equations to obtain the magnetic field. The building blocks of this methodology, introduced in different contexts [1, 19, 20], are here combined and applied for the first time for fully three-dimensional ferrofluid simulations. The mesh adaptive methodology allow us to compute the flow and the droplet deformation with unprecedented resolution and accuracy. However, our simulations are limited to small capillary numbers (see Section 2.2 for a description of all dimensionless groups) and we consider only a small number of points in parameter space due to the enormous computational cost for each simulation to steady state (weeks of CPU and some of them months). Nevertheless, our study shows clearly the significant field-induced distortions of the ferrofluid droplet and the effect of the field on the direction of maximal deformation and on the overall dynamics towards steady state. In particular, we find that for moderate to strong fields (relative to surface tension) and weak flows, the field-induced droplet distortion can be quite significant. Moreover, the droplet deformation in the vorticity direction can be of the same order as that occurring in the shear plane. This and other results in this study highlight the importance of the three-dimensional character of this significant rheological problem.

The paper is organized as follows: In Section 2, we present the governing equations, the immersed boundary formulation, and the nondimensional form of the equations. The physical parameters used in this study, which correspond to an actual PDMS ferrofluid droplet suspended in viscous medium, are given at the end of this section. We summarize the numerical methodology in Section 3. More details of the numerical discretization appear in Appendix A. Section 4 is devoted to the numerical results, including a validation and a comparison with an asymptotic small deformation theory. Considerations about a verification of the numerical accuracy are presented in Appendix B. A resolu-