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Numerical Method of Profile Reconstruction for a Periodic Transmission Problem from Single-Sided Data

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Abstract. We are concerned with the profile reconstruction of a penetrable grating from scattered waves measured above the periodic structure. The inverse problem is reformulated as an optimization problem, which consists of two parts: a linear severely ill-posed problem and a nonlinear well-posed problem. A Tikhonov regularization method and a Landweber iteration strategy are applied to the objective function to deal with the ill-posedness and nonlinearity. We propose a self-consistent method to recover a potential function and an approximation of grating function in each iterative step. Some details for numerical implementation are carefully discussed to reduce the computational efforts. Numerical examples for exact and noisy data are included to illustrate the effectiveness and the competitive behavior of the proposed method.

AMS subject classifications: 35R30, 78A46, 78M50

Key words: Profile reconstruction, optimization method, Tikhonov regularization, periodic transmission problem.

1 Introduction

Direct and inverse scattering by periodic structures has many applications in micro-optics, where periodic structures are often called diffraction gratings. During the last two decades, direct scattering problems for diffraction gratings received considerable attention in the applied mathematics and physics, and were studied extensively by ether integral equation methods [27] or variational methods [7]. We refer to the monograph [28] for a good introduction to the mathematical theories of electromagnetic diffraction by periodic structures. Numerical methods for direct scattering problems can be found in [4, 5, 14, 15, 21, 30].

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Inverse diffraction grating problem has been an active research field in the past several decades. A great deal of applications are associated with the determination of the nature of a periodic structure from a measured diffracted field. Uniqueness theories and stability analysis of the inverse diffraction grating could be found in [1,3,6,9–11,22,24]. Numerical methods on the inverse problem for impenetrable gratings have been extensively studied. In [23], Ito and Reitich presented a high-order perturbation approach based on methods of variation of boundaries. Arens and Kirsch [2] applied the factorization method to scattering by a periodic surface. A two-step algorithm was discussed by Bruckner and Elschner in [12]. Based on the shape derivatives with respect to the variations of the boundary, Hettlich [21] proposed two iterative regularization methods. An efficient continuation method was developed by Bao *et al.* in [8] using multiple frequency data. Elschner [18] presented a numerical algorithm by combining finite element methods and optimization techniques. Another important application of diffractive optics is to design a grating structure that generates some specified far-field patterns [16,17,19,20].

Arising from increasing demand in fast non-destructive test, the reconstruction of periodic penetrable structures is attracting extensive attention of mathematicians and engineers. For two-layered medium problem, the factorization method and the linear sampling method were studied in [25] and [33]. An iterative method was presented for shallow penetrable gratings by Malcolm and Nicholls in [26]. To recover the support of a periodic medium which separates the whole space into three parts, sampling methods were adopted by Yang *et al.* in [31], and by Sun and Zhang in [29]. The factorization method was considered recently in [32]. Some quantitative methods were also studied to reconstruct transmission grating profiles. Bruckner and Elschner [13] developed an optimization method by using two-sided scattered electromagnetic field. By using only single-sided measured data, a finite element method was presented by Zhang and Sun in [34]. In the current paper, we are concerned with an optimization method to recover the grating profile from the single-sided measurement. In this method, a cost functional is properly presented and numerically solved by executing a two-step procedure.

The rest of this paper is outlined as follows. In Section 2, we give a mathematical model of the scattering by a penetrable grating problem, and introduce some necessary notations. In Section 3, we describe an inverse problem for the penetrable diffraction grating, and reformulate it as an optimization problem. Section 4 is devoted to the numerical implementation of the proposed method in detail. In Section 5, we present several representative numerical examples to demonstrate the competitive behavior of our method. Some concluding remarks are given in Section 6.

2 Model problem

In this section, we outline the mathematical model of the penetrable diffraction grating and introduce some necessary notations. We consider a periodic surface within one period of 2π. The surface is assumed to be invariant in the z-direction. Let the cross section