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Recursive POD Expansion for the Advection-Diffusion-Reaction Equation

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Abstract. This paper deals with the approximation of advection-diffusion-reaction equation solution by reduced order methods. We use the Recursive POD approximation for multivariate functions introduced in [5] and applied to the low tensor representation of the solution of the reaction-diffusion partial differential equation. In this contribution we extend the Recursive POD approximation for multivariate functions with an arbitrary number of parameters, for which we prove general error estimates. The method is used to approximate the solutions of the advection-diffusion-reaction equation. We prove spectral error estimates, in which the spectral convergence rate depends only on the diffusion interval, while the error estimates are affected by a factor that grows exponentially with the advection velocity, and are independent of the reaction rate if this lives in a bounded set. These error estimates are based upon the analyticity of the solution of these equations as a function of the parameters (advection velocity, diffusion, reaction rate). We present several numerical tests, strongly consistent with the theoretical error estimates.

AMS subject classifications: 65D15, 65M70, 74F10

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1 Introduction

Physical and real phenomena are often described by large scale problems for which a low dimensional approximation turns out to be essential. Therefore, in the last decades,

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ROMs, known as Karhunen-Loève expansion (KLE), [21], have gained popularity. Such techniques are also known as Proper Orthogonal Decomposition (POD), [8], in mechanical computation, Principal Component Analysis (PCA) in statistics, [7, 18], and Singular Value Decomposition (SVD) in linear algebra, [16]. The POD is a very general information compression technique which finds its natural applications in a wide variety of fields, such as in digital image compression, bioinformatics, signal processing and resolution of PDEs, [3,4,6,8,19,25] and also the recent work of Cohen et al related to the numerical analysis of the convergence of polynomial approximation of parametric elliptic PDE's [11,12]. Hence, the problem of the tensor representation of multivariate phenomena, by means of the KLE, is a challenging computational issue.

Recently, techniques as the Proper Generalized Decomposition (PGD), [17], which gives a suitable approximation of multivariate functions by low dimensional varieties, or as the Higher Order SVD (HOSVD), [13, 14], have been developed. The latter provides a multilinear generalization of the best rank-l approximation problem for matrices obtained by truncating the SVD. In fact, the HOSVD enables us to perform a low dimensional approximation of tensors as the SVD allows to approximate bivariate data. Also, a recent variant of the HOSVD is the Truncated Sequential HOSVD (ST-HOSVD), introduced in [26], that consists in sequentially performing a SVD analysis of the successive modes. The fact that each step for the construction of the ST-HOSVD separates two variables by the SVD procedure allows to exactly compute the truncation error in terms of the remaining singular values.

This paper focuses on the study of the low tensor representation of the solution of the reaction-diffusion partial differential equation. The Recursive POD (RPOD) was proposed in [5], as a procedure to build low dimensional representation of trivariate functions. It was applied to the diffusion-reaction equation, for which an exponential rate of convergence, depending only on the diffusion rate, was proved. In the present paper we study the extension of the RPOD to approximate the solution of the advection-diffusion-reaction equations. To do this, we start first by proving a more general result of convergence of the RPOD expansion to approximate parametric analytic function in L^2 space. Then the analysis of the RPOD expansion of solution of reaction-diffusion partial differential equation turns to be a particular case where the parameters are advection velocity –assumed constant–, diffusion, reaction rate and space-time coordinates. We prove a spectral rate of convergence, that depends only on the diffusion rate much as in [5] for the convection-reaction equation. However here this rate is affected by a factor that exponentially grows with the Péclet number, and is independent on the reaction rate if this lives in a bounded set.

We also present some additional numerical tests that show the suitability of the RPOD to approximate analytic functions depending on a moderate number of parameters, with an exponential convergence rate.

The outline of the paper is as follows. Section 2 focuses on the presentation of the RPOD expansion to multivariate functions with an arbitrary number of parameters. In section 3 we study the application of the RPOD expansion to approximate parametric