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Fully-Coupled Multi-Physical Simulation with Physics-Based Nonlinearity-Elimination Preconditioned Inexact Newton Method for Enhanced Oil Recovery

Huiying Tang[1], Shihao Wang[2,*], Congbin Yin[3], Yuan Di[4], Yu-Shu Wu[2] and Yonghong Wang[5]

[1] *State Key Laboratory of Oil and Gas Reservoir Geology and Exploitation, Southwest Petroleum University, Chengdu, P.R. China.*
[2] *Petroleum Engineering Department, Colorado School of Mines, Golden, CO 80401, USA.*
[3] *Chuanqing Downhold Service, Chengdu, P.R. China.*
[4] *Department of Energy and Resources Engineering, Peking University, Beijing 100871, P.R. China.*
[5] *CNPC Engineering Technology R&D Company Limited, Houston, Texas 77042, USA.*

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Abstract. In this paper, we introduce a physics-based nonlinear preconditioned Inexact Newton Method (INB) for the multiphysical simulation of fractured reservoirs. Instead of solving the partial differential equations (PDE) exactly, Inexact Newton method finds a direction for the iteration and solves the equations inexactly with fewer iterations. However, when the equations are not smooth enough, especially when local discontinuities exits, and when proper preconditioning operations are not adopted, the Inexact Newton method may be slow or even stagnant.

As pointed out by Keyes et al. [1], multi-physical numerical simulation faces several challenges, one of which is the local-scale nonlinearity and discontinuity. In this work, we have proposed and studied a nonlinear preconditioner to improve the performance of Inexact Newton Method. The nonlinear preconditioner is essentially a physics-based strategy to adaptively identify and eliminate the highly nonlinear zones.

The proposed algorithm has been implemented into our fully coupled, fully implicit THM reservoir simulator (Wang et al. [2, 3]) to study the effects of cold water injection on fractured petroleum reservoirs. The results of this work show that after the implementation of this nonlinear preconditioner, the iterative solver has become significantly more robust and efficient.

AMS subject classifications: 60-08

Key words: Physics-based nonlinearity-elimination, Inexact Newton method, thermal-hydraulic-mechanical simulation, restricted additive Schwarz approach, parallel reservoir simulation.

*Corresponding author. *Email addresses:* gohych@gmail.com (S. Wang), appythy@gmail.com (H. Tang), cyin@chuanqing.com (C. Yin), diyuan@mech.pku.edu.cn (Y. Di), ywu@mines.edu (Y.-S. Wu), yonghong.wang@cnpcusa.com (Y. Wang)

1 Introduction

Newton's method is a broad range of iterative methods for nonlinear programming. Starting from an initial guess, Newton's method computes the search direction based on the gradient of the given function. Once the search direction is determined, the searching step can be calculated by multiple approaches, including line-search and directly computing from residuals (Newton-Raphson method).

Exact Newton's method solves the problem iteratively until the residual is smaller than a pre-set criteria. In petroleum reservoir simulation, exact Newton-Raphson (NR) method is widely adopted. Inexact Newton's method (IN), on the other hand, solves the problem inexactly. Compared to exact Newton's method, IN requires less computation time, but may have poorer numerical stability. The Inexact Newton Method with Backtracking (INB) is an appealing approach for large scale numerical simulation. INB solves a nonlinear system approximately within each iteration. It can save much computational time spent that would otherwise be used for the linear solver. However, one of the drawbacks of the INB algorithm is that it is not as robust as Newton-Raphson algorithm (NR) in certain cases, as reported by other researchers [4–6]. For INB algorithm, it has been proven by Kelley [7] that if the target nonlinear equation is continuously differentiable and there is a limit point at which the Jacobian matrix is nonsingular, then the INB will converge at that limit point. With the existence of local discontinuities, the target equation is no longer continuously differentiable and the more the discontinuities, the farther the system is away from a continuously differential condition. Therefore as the number of local discontinuities increases, INB may suffer from convergence problems. Globalization techniques [8, 9] help improve the numerical performance of IN, but they cannot fully resolve the convergence issue, as reported by Knoll and Keyes [10].

For several types of reservoir simulation problems, such as water-oil displacement, nonlinearity exists only in a small portion of the entire field. This local nonlinearity will dramatically slow down the convergence of INB solvers. For the INB method, if the local nonlinearity is too high, the iteration will tend to be stagnant. Therefore, to guarantee the robustness of the INB, a nonlinear preconditioner is typically required, such as the Additive Schwarz Preconditioned Inexact Newton (ASPIN), as proposed by Cai and Keyes [11]. Additive Schwarz (AS) [12] method is essentially a domain decomposition approach. It subdivides the problem domain into several sub-domains and solves the sub-space problem separately in parallel. For non-overlapping AS, there is no communication among subdomains. For overlapping AS, neighboring subdomains exchange boundary conditions with each other. Cai and Sarkis [13] brought out the Restricted Additive Schwarz method, which has less communication among processors and better the numerical stability.

Recently, ASPIN has been applied to groundwater and oil simulation problems by (Skogestad et al. [14] and Liu et al. [15], respectively. ASPIN has been shown to be able to improve the convergence rate for highly heterogeneous reservoir simulation problems. As far as we know, there have very few attempts to adopt the nonlinearity elimination