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FDMs for the PDEs of Option Pricing Under DEV Models with Counterparty Risk

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Abstract. In this paper we study the option pricing problem under dynamic elasticity of variance (DEV) model with counterparty risk. The counterparty risk induces a drop in the asset price and the asset can still be traded after this default time. There are no explicit solutions for the value function of the options. The value functions are governed by two joint partial differential equations (PDEs) which are connected at the default time. The PDEs are discretized by the finite difference methods (FDMs) and the second-order convergence rates both in time and space are derived. Numerical examples are carried out to verify the convergence results.

AMS subject classifications: 35R35, 91G20, 91G60, 91G80

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1. Introduction

In this paper, we consider a financial market model with a risky asset (stock) whose price follows the DEV model with counterparty risk. The dynamics of the stockis affected by the possibility of the counterparty defaulting. However, this stock still exists and can be traded after such default.

Assume $(\Omega, \mathcal{G}, \mathbb{P})$ is a complete probability space satisfying the usual conditions. Let $(W_t)_{t\in[0,T]}$ be a Brownian motion with horizon $T < \infty$ on the probability space $(\Omega, \mathcal{G}, \mathbb{P})$ and denote by $\mathbb{F} = (\mathcal{F}_t)_{t\in[0,T]}$ the natural filtration of W. Let τ, an almost surely nonnegative random variable on $(\Omega, \mathcal{G}, \mathbb{P})$, represent the default time. Then $(\mathcal{H}_t)_{t\in[0,T]}$ is defined by $\mathcal{H}_t := \sigma(H_u : u \leq t)$, where $H_t := \mathbf{1}_{\{\tau\leq t\}}$ which equals 0 if $\tau > t$ and 1 otherwise, and denote $\mathbb{H} = (\mathcal{H}_t)_{t\in[0,T]}$. Denote $\mathbb{G} = (\mathcal{G}_t)_{t\in[0,T]}$ the progressively

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enlarged filtration $\mathbb{G} = \mathbb{F} \vee \mathbb{H}$, representing the structure of information available for the investors over $[0, T]$. The stock price process S_t is given by the following stochastic differential equation:

$$dS_t = \mu_t S_{t-} dt + \delta_t S_{t-}^{f(t)/2} dW_t - \gamma_t S_{t-} dH_t, \qquad 0 \le t \le T, \tag{1.1}$$

where μ_t, δ_t and γ_t are $\mathbb{G}$-predictable processes, $f(t)$ is a dynamic function with respect to time t. The process γ_t represents the percentage loss or gain on the stock price induced by the defaults of the counterparty. If f is a constant, then stock price model (1.1) becomes the constant elasticity of variance (CEV) model with counterparty risk.

According to Mansuy and Yor [13],any $\mathbb{G}$-predictable process φ_t can be written as

$$\varphi_t = \varphi_t^F \mathbf{1}_{\{t<\tau\}} + \varphi_t^d(\tau) \mathbf{1}_{\{t \ge \tau\}}, \qquad 0 \le t \le T, \tag{1.2}$$

with φ_t^F is $\mathbb{F}$-adapted and $\varphi_t^d(\vartheta)$is ϑ-measurable and $\mathbb{F}$-adapted. Therefore, the dynamic of stock price under physical measure $\mathbb{P}$ can be rewritten as

$$dS_t^F = \mu_t^F S_t^F dt + \delta_t^F (S_t^F)^{f(t)/2} dW_t, \qquad 0 \le t < \tau, \tag{1.3a}$$

$$dS_t^d(\tau) = \mu_t^d(\tau) S_t^d(\tau) dt + \delta_t^d(\tau) \big(S_t^d(\tau)\big)^{f(t)/2} dW_t, \qquad \tau < t \le T, \tag{1.3b}$$

$$S_\tau^d(\tau) = S_\tau^F \big(1 - \gamma_\tau^F\big), \tag{1.3c}$$

where μ_t^F, δ_t^F, S_t^F and γ_τ^F are $\mathbb{F}$-adapted, $\mu_t^d(\vartheta)$, $\delta_t^d(\vartheta)$ and $S_t^d(\vartheta)$ are ϑ-measurable and $\mathbb{F}$-adapted. It should be noted that S_τ^F implies $S_\tau^F = \lim_{t \to \tau^-} S_t^F$. We, for simplicity, assume that

$$\mu_t^F = \mu_1, \qquad \delta_t^F = \delta_1, \qquad \mu_t^d(\tau) = \mu_2, \qquad \delta_t^d(\tau) = \delta_2, \qquad \gamma_\tau^F = \gamma,$$

where μ_1, δ_1, μ_2, δ_2 are nonnegative constants and γ, $(\gamma < 1)$ is a given random variable. In practice, we may assume γ is a discrete random variable to simplify the computation, in what follows, we further assume that γ takes value γ_i with probability p_i for $i = 1, 2, 3$, where $0 < \gamma_1 < 1$ (loss), $\gamma_2 = 0$ (no change), and $\gamma_3 < 0$ (gain). Moreover γ, τ, W_t are independent and τ is an exponential random variable with parameter λ. For more details related to counterparty risk model, we refer to Jiao and Pham [11]. From (1.3a)-(1.3c), it is obvious that if $\gamma = 0$ then there is no jump of stock price at time τ, and this model becomes a simple regime switching model.

At the events of crashes the volatility changes turbulently in short period and it is hard to find an effective process or distribution to express this phenomenon. On the other hand, although some small sizes of rebounding in crash, the stock prices would be sharply decreasing, which leads to the fact that the volatility of stock prices becomes increasing greatly. But for the CEV model, the volatility term has the same monotonic trend as the stock prices when its exponent is greater than 2. This makes a contradiction. To overcome these difficulties, Fan et al. [6] extend the CEV model to the DEV model, whose the volatility term of stock is a compound function based on it prices to the power of a nonparametric function. In their works, the authors also