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A Second-order Hyperbolic Chemotaxis Model

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Abstract. In this paper, we study a hyperbolic type chemotaxis model in one space dimension. We assume the speed is constant, the production and degradation of the external signal s is given by $-\beta s+f(u^+ +u^-)$, where $f(u^+ +u^-)$ is the general form and u^+,u^- depend on s. The existence of the weak solution of the model considered in the paper is obtained by the method of characteristics and the contraction mapping principle.

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1 Introduction

Individuals movement is a widespread phenomenon in biological systems. They are looking for food, enduring starvation conditions, exploring new regions and avoiding predators by movement. Here we consider biological species which sense an external stimulus, produced by themselves, and move towards it. This behavior is called positive taxis. According to the type of the external stimulus, positive taxis are divided into chemotaxis, haptotaxis, aerotaxis and others.

In this paper we will consider chemotaxis, which means that the external signal is a chemical nutrient which could be recognized by receptors of the individual particles and describes the response of the individuals to an external chemicals or its gradient.

In 1970, the first mathematical model describing chemotaxis was introduced by Keller and Segel [1], but with no reasonable limit to the speed of particles. In 1977, a hyperbolic model based on Goldstein-Kac [2] model in one dimension was introduced by Segel [3], which is

$$u_t^+ +\gamma u_x^+ =\mu(u^- -u^+), \tag{1.1}$$

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$$u_t^- - \gamma u_x^- = \mu(u^+ - u^-), \tag{1.2}$$

where the total population density $u(t,x)$ can be split into densities for right /left moving part of the population, $u^\pm$, respectively. Of course $u=u^++u^-$. The individuals move with constant speed $\gamma>0$ and the constant turning rate $\mu>0$.

In general the speed and the turning rates of individuals depend not only on the magnitude of an external signal $s(t,x)$ but on temporal and spatial variations $s_t(t,x)$ and $s_x(t,x)$ as well (see [4–7]). Depending on the situation at hand, the signal can be produced and decay in time. This will be descried by a function $f(s,u^++u^-)$. Then a modification of the Goldstein-Kac model gives the following hyperbolic model for chemosensitive movement in one space dimension:

$$u_t^+ + \gamma(s,s_t,s_x)u_x^+ = -\mu^+(s,s_t,s_x)u^+ + \mu^-(s,s_t,s_x)u^-, \tag{1.3}$$

$$u_t^- - \gamma(s,s_t,s_x)u_x^- = \mu^+(s,s_t,s_x)u^+ - \mu^-(s,s_t,s_x)u^-, \tag{1.4}$$

$$\tau s_t = Ds_{xx} + f(s,u^++u^-), \qquad \tau,D>0. \tag{1.5}$$

Here the rates $\mu^\pm$ are right/left turning rates, respectively.

Later, combing the hyperbolic chemotaxis model (1.3)-(1.5) and the classical Keller-Segel [1] model and choosing appropriate parameter functions $\mu^\pm$ and γ, where $\mu^\pm$ depend on s and s_x, γ is constant, Hillen [8] get the local and global existence of solution in L^∞. To achieve an abstract existence result for dependence on s_t , a more detailed analysis is required. Without s_t dependence the preservation of total population size suffices to show existence of weak solution in L^∞. To control s_t stronger pre-assumptions are required. If the speed depends on s or it's gradient we expect the formation of steep gradients. This case has been considered in [10].

The external signal satisfy the parabolic equation in the original problem according to that its spread speed is infinite. Later, Wang and Wu [12] consider the case of first order hyperbolic model, in which the external s is given by the following equation:

$$\tau s_t = Ds_x + f(s,u^++u^-). \tag{1.6}$$

As we all know, the external signal transmission way is varied. Chen and Wu [14] consider the external signal to meet the second order hyperbolic situation. Jiang [15] also consider the similar problem, which means that s is given by the following equation:

$$s_{tt} = s_{xx} + f(s,u^++u^-), \tag{1.7}$$

where $f(s,u^++u^-) = -\beta s + \alpha(u^++u^-), \beta>0, \alpha>0$.

Now, we consider the case that the production and degradation of the external signal s is given by $-\beta s + f(u^++u^-)$, where $f(u^++u^-)$ is the general form, and u^+,u^- depend on s. Then we have the following equation:

$$s_{tt} = s_{xx} - \beta s + f(u^++u^-), \quad \beta>0. \tag{1.8}$$