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Solvability for a Coupled System of Fractional p-Laplacian Differential Equations at Resonance

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Abstract: In this paper, by using the coincidence degree theory, the existence of solutions for a coupled system of fractional p-Laplacian differential equations at resonance is studied. The result obtained in this paper extends some known results. An example is given to illustrate our result.
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1 Introduction

In this paper, by using the coincidence degree theory, we discuss the existence of solutions to a coupled system of fractional p-Laplacian differential equations at resonance:

$$\begin{cases} D_{0+}^{\beta}\phi_{p_1}(D_{0+}^{\alpha}u(t)) = f_1(t,\ u(t),\ v(t),\ D_{0+}^{\alpha}u(t),\ D_{0+}^{\alpha}v(t)), & 0<t<1,\\ D_{0+}^{\beta}\phi_{p_2}(D_{0+}^{\alpha}v(t)) = f_2(t,\ u(t),\ v(t),\ D_{0+}^{\alpha}u(t),\ D_{0+}^{\alpha}v(t)), & 0<t<1,\\ u(0)=D_{0+}^{\alpha}u(0)=0, \qquad u(1)=\sum\limits_{i=1}^{n_1}A_iu(\epsilon_i), & \\ D_{0+}^{\gamma}\phi_{p_1}(D_{0+}^{\alpha}u(t))|_{t=1} = \sum\limits_{i=1}^{n}a_iD_{0+}^{\gamma}\phi_{p_1}(D_{0+}^{\alpha}u(t))|_{t=\xi_i}, & \\ v(0)=D_{0+}^{\alpha}v(0)=0, \qquad v(1)=\sum\limits_{i=1}^{m_1}B_iv(\sigma_i), & \\ D_{0+}^{\delta}\phi_{p_2}(D_{0+}^{\alpha}v(t))|_{t=1} = \sum\limits_{i=1}^{m}b_iD_{0+}^{\delta}\phi_{p_2}(D_{0+}^{\alpha}v(t))|_{t=\eta_i}, & \end{cases} \tag{1.1}$$

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where $1 < \alpha, \beta \le 2$, and $3 < \alpha + \beta \le 4$; $0 < \gamma,\ \delta \le \beta - 1$; $\phi_{p_i}(x) = |x|^{p_i-2}x$, $p_i > 1$, $\phi_{q_i} = \phi_{p_i}^{-1}$, $\dfrac{1}{p_i} + \dfrac{1}{q_i} = 1$, $i = 1, 2$; $0 < \epsilon_1 < \epsilon_2 < \cdots < \epsilon_{n_1} < 1$, $0 < \sigma_1 < \sigma_2 < \cdots < \sigma_{m_1} < 1$, $0 < \xi_1 < \xi_2 < \cdots < \xi_n < 1$, $0 < \eta_1 < \eta_2 < \cdots < \eta_m < 1$; $A_r,\ a_j,\ B_k,\ b_l \in \mathbf{R}$, $r = 1, 2, \cdots, n_1$, $j = 1, 2, \cdots, n$, $k = 1, 2, \cdots, m_1$, $l = 1, 2, \cdots, m$. D^α, D^β, D^γ and D^δ are the standard Riemann-Liouville fractional derivatives.

In this paper, we always suppose that the following conditions hold.

(H_1) $\sum\limits_{i=1}^{n_1} A_i\epsilon_i^{\alpha-1} = 1$, $\sum\limits_{i=1}^{m_1} B_i\sigma_i^{\alpha-1} = 1$, $\sum\limits_{i=1}^{n} a_i\xi_i^{\beta-\gamma-1} = 1$, $\sum\limits_{i=1}^{m} b_i\eta_i^{\beta-\delta-1} = 1$, $\sum\limits_{i=1}^{n_1} A_i\epsilon_i^{\alpha} \ne 1$, $\sum\limits_{i=1}^{m_1} B_i\sigma_i^{\alpha} \ne 1$, $\sum\limits_{i=1}^{n} a_i\xi_i^{\beta-\gamma} \ne 1$, $\sum\limits_{i=1}^{m} b_i\eta_i^{\beta-\delta} \ne 1$.

(H_2) f_i: $[0,1] \times \mathbf{R}^4 \to \mathbf{R}$ satisfied Carathéodory conditions, $i = 1, 2$, that is,

(i) $f(\ \cdot\ ;\ x_1, x_2, x_3, x_4)$: $[0,1] \to \mathbf{R}$ is measurable for all $(x_1, x_2, x_3, x_4) \in \mathbf{R}^4$;

(ii) $f(t;\ \cdot\ ,\ \cdot\ ,\ \cdot\ ,\ \cdot\ )$: $\mathbf{R}^4 \to \mathbf{R}$ is continuous for a.e. $t \in [0,1]$;

(iii) for each compact set $\mathcal{K} \subset \mathbf{R}^4$ there is a function $\varphi_{\mathcal{K}} \in L^\infty[0,1]$ such that

$$|f(t, x_1, x_2, x_3, x_4)| \le \varphi_{\mathcal{K}}(t)$$

for a.e. $t \in [0,1]$ and all $(x_1, x_2, x_3, x_4) \in \mathcal{K}$.

The existence of solutions for boundary value problem of integer order differential equations at resonance has been studied by many authors (see [1]–[10] and references cited therein). Since the extensive applicability of fractional differential equations (see [11] and [12]), recently, more and more authors pay their close attention to the boundary value problems of fractional differential equations (see [13]–[20]). In papers [13] and [14], the existence of solutions to coupled system of fractional differential equations at nonresonance has been given. In papers [15] and [16], the solvability of fractional differential equations at resonance has been investigated.

Paper [16] investigates the following coupled system of fractional differential equations at resonance:

$$\begin{cases} D_{0^+}^{\alpha} u(t) = f_1(t,\ u(t),\ v(t)), & 0 < t < 1, \\ D_{0^+}^{\beta} v(t) = f_2(t,\ u(t),\ v(t)), & 0 < t < 1, \\ u(0) = 0, \quad D_{0^+}^{\gamma} u(t)|_{t=1} = \sum\limits_{i=1}^{n} a_i D_{0^+}^{\gamma} u(t)|_{t=\xi_i}, & \\ v(0) = 0, \quad D_{0^+}^{\delta} v(t)|_{t=1} = \sum\limits_{i=1}^{m} b_i D_{0^+}^{\delta} v(t)|_{t=\eta_i}, & \end{cases} \tag{1.2}$$

where $1 < \alpha, \beta \le 2$, $0 < \gamma \le \alpha - 1$, $\delta \le \beta - 1$; $0 < \xi_1 < \xi_2 < \cdots < \xi_n < 1$, $0 < \eta_1 < \eta_2 < \cdots < \eta_m < 1$; $\sum\limits_{i=1}^{n} a_i\xi_i^{\beta-\gamma-1} = 1$, $\sum\limits_{i=1}^{m} b_i\eta_i^{\beta-\delta-1} = 1$. By using the coincidence degree theory due to Mawhin and constructing suitable operators, the existence of solutions for (1.2) is obtained.

In the past few decades, in order to meet the demands of research, the p-Laplacian equation is introduced in some BVP, such as [17] and [18].