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Planar Cubic G^1 Hermite Interpolant with Minimal Quadratic Oscillation in Average

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Abstract: In this paper we apply a new method to choose suitable free parameters of the planar cubic G^1 Hermite interpolant. The method provides the cubic G^1 Hermite interpolant with minimal quadratic oscillation in average. We can use the method to construct the optimal shape-preserving interpolant. Some numerical examples are presented to illustrate the effectiveness of the method.

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1 Introduction

As we know, the planar cubic Hermite interpolant includes two cases of C^1 continuity and G^1 continuity. The cubic C^1 Hermite interpolant can be described as follows:

$$\boldsymbol{b}(t) := \boldsymbol{b}_i(t) = \boldsymbol{p}_i B_0^3(t) + \left(\boldsymbol{p}_i + \frac{1}{3}\boldsymbol{m}_i\right) B_1^3(t) + \left(\boldsymbol{p}_{i+1} - \frac{1}{3}\boldsymbol{m}_{i+1}\right) B_2^3(t) + \boldsymbol{p}_{i+1} B_3^3(t), \quad (1.1)$$

where $i = 0, 1, \cdots, n-1$, $\boldsymbol{p}_i$ and $\boldsymbol{p}_{i+1}$ are two end points together with two tangent vectors $\boldsymbol{m}_i$ and $\boldsymbol{m}_{i+1}$,

$$B_k^3(t) = \frac{3!}{k!(3-k)!} t^k (1-t)^{3-k}, \qquad k = 0, 1, 2, 3$$

are cubic Bernstein polynomials. The cubic G^1 Hermite interpolant can be described as follows:

$$\boldsymbol{b}(t) := \boldsymbol{b}_i(t) = \boldsymbol{p}_i B_0^3(t) + \left(\boldsymbol{p}_i + \frac{1}{3}\alpha_{i,0}\boldsymbol{d}_i\right) B_1^3(t) + \left(\boldsymbol{p}_{i+1} - \frac{1}{3}\alpha_{i,1}\boldsymbol{d}_{i+1}\right) B_2^3(t) + \boldsymbol{p}_{i+1} B_3^3(t), \quad (1.2)$$

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where $i = 0, 1, \cdots, n-1$, $\boldsymbol{d}_i$ and $\boldsymbol{d}_{i+1}$ are unit tangent directions of the two end points, $\alpha_{i,0}$ and $\alpha_{i,1}$ are positive numbers.

For the cubic C^1 Hermite interpolant expressed in (1.1), the tangent vectors $\boldsymbol{m}_i$, $\boldsymbol{m}_{i+1}$ are generally required to be given in advance. But in practical applications the tangent vectors are rarely available. In most cases, minimization an appropriate functional is applied to determine the tangent vectors. As it was proposed in [1] and [2], one can determine the tangent vectors by minimizing

$$J_k = \int_0^1 \|\boldsymbol{b}^{(k)}(t)\|^2 \mathrm{d}t, \qquad k = 0, 1, 2, 3. \tag{1.3}$$

Different to (1.3), in [3] and [4], the method was presented by minimizing

$$I_k = \int_0^1 \|\boldsymbol{b}^{(k)}(t) - \boldsymbol{L}^{(k)}(t)\|^2 \mathrm{d}t, \qquad k = 0, 1, \tag{1.4}$$

where

$$\boldsymbol{L}(t) := \boldsymbol{L}_i(t) = (1-t)\boldsymbol{p}_i + t\boldsymbol{p}_{i+1}.$$

Particularly, the method by minimizing I_0 expressed in (1.4) provides the interpolant with minimal quadratic oscillation in average, and the method by minimizing I_1 expressed in (1.4) provides the interpolant with minimal derivative oscillation.

For the cubic G^1 Hermite interpolant in (1.2), the unit tangent directions $\boldsymbol{d}_i$, $\boldsymbol{d}_{i+1}$ are generally given while the numbers $\alpha_{i,0}$ and $\alpha_{i,1}$ are usually used as free parameters. Although there are several approaches to choose suitable free parameters, the common ways to determine the free parameters are achieved by minimizing J_k that have been applied to the cubic C^1 Hermite interpolant. In particular, the method by minimizing J_2 provides the interpolant with minimal strain energy (see [5] and [6]), and the method by minimizing J_3 provides the interpolant with minimal curvature variation (see [7]–[9]).

A natural idea arises: maybe we can use the methods by minimizing I_k to choose the parameters $\alpha_{i,0}$ and $\alpha_{i,1}$ for the cubic G^1 Hermite interpolant, since the methods have been applied to the cubic C^1 Hermite interpolant. With this in mind, we consider here the problem of determining the parameters $\alpha_{i,0}$ and $\alpha_{i,1}$ by minimizing I_0 that provides the cubic C^1 Hermite interpolant with minimal quadratic oscillation in average. Because the piecewise linear interpolant $\boldsymbol{L}(t)$ is the simplest shape-preserving interpolant, the cubic Hermite interpolant obtained by minimizing I_0 is almost flat-shaped. This means we can use the method by minimizing I_0 to construct the optimal shape-preserving interpolant.

The remainder of this paper is organized as follows. We present the computing method for determining the parameters $\alpha_{i,0}$ and $\alpha_{i,1}$ by minimizing I_0 in Section 2. In Section 3 we show some numerical examples in comparison with the method of curvature variation minimization.

2 The Computing Method

Because the cubic G^1 Hermite interpolant can be studied locally, we only need to consider a piece of curve $\boldsymbol{b}_i(t)$. According to (1.4), the quadratic oscillation in average of $\boldsymbol{b}_i(t)$ can