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Further Results on Meromorphic Functions and Their nth Order Exact Differences with Three Shared Values

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Abstract: Let $E(a, f)$ be the set of a-points of a meromorphic function $f(z)$ counting multiplicities. We prove that if a transcendental meromorphic function $f(z)$ of hyper order strictly less than 1 and its nth exact difference $\Delta_c^n f(z)$ satisfy $E(1, f) = E(1, \Delta_c^n f)$, $E(0, f) \subset E(0, \Delta_c^n f)$ and $E(\infty, f) \supset E(\infty, \Delta_c^n f)$, then $\Delta_c^n f(z) \equiv f(z)$. This result improves a more recent theorem due to Gao *et al.* (Gao Z, Kornonen R, Zhang J, Zhang Y. Uniqueness of meromorphic functions sharing values with their nth order exact differences. *Analysis Math.*, 2018, https://doi.org/10.1007/s10476-018-0605-2) by using a simple method.

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1 Introduction and Main Results

We assume that the reader is familiar with the fundamental results and standard notations of Nevanlinna's theory, as found in [1] and [2], such as the characteristic function $T(r, f)$ of a meromorphic function $f(z)$. Notation $S(r, f)$ means any quantity such that $S(r, f) = o(T(r, f))$ as $r \to \infty$ outside of a possible set of finite logarithmic measure. Given one value $a \in \mathbf{C} \cup \{\infty\}$ and two meromorphic functions $f(z)$ and $g(z)$, we say that f and g share a CM (IM) when f and g have the same a-points counting multiplicities (ignoring multiplicities). Denote by $E(a, f)$ the set of all zeros of $f - a$, where a zero with multiplicity m is counted m times in $E(a, f)$, then f and g share a CM provided that $E(a, f) = E(a, g)$. Moreover, if $E(a, f) \subset E(a, g)$, then we say that f has a partially shared value a CM with g.

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About ten years ago, Halburd and Korhonen[3],[4], Chiang and Feng[5] established the difference analogue of Nevanlinna's theory for finite order meromorphic functions, independently. Later, Halburd *et al.*[6] showed that it is still valid for meromorphic functions of hyper-order strictly less than 1. So far, it has been a most useful tool to study the uniqueness problems between meromorphic functions $f(z)$ and their shifts $f(z+c)$ or the nth exact differences $\Delta_c^n f(z)$ $(n \geq 1)$. For some related results in this topic, we refer the readers to reference [7]–[11] and so on.

In 2013, Chen and Yi[7] proved a uniqueness theorem for a meromorphic function $f(z)$ and its first order exact difference $\Delta_c f(z)$ with three distinct shared values CM, which had been improved by Zhang and Liao[11] in 2014, Lü F and Lü W R[10] in 2016 and Chen[12] in 2018. In this direction, Gao *et al.*[13] obtained the following uniqueness theorem concerning the nth exact difference more recently.

Theorem 1.1[13] *Let f be a transcendental meromorphic function of hyper order strictly less than 1 such that $\Delta_{c=1}^n f(z) \not\equiv 0$. If $f(z)$ and $\Delta_{c=1}^n f(z)$ share three distinct periodic functions a, b, $c \in \hat{\mathcal{S}}(f)$ with period 1 CM, then $\Delta_{c=1}^n f(z) \equiv f(z)$.*

Here, the notation $\hat{\mathcal{S}}(f)$ means $\mathcal{S}(f) \cup \{\infty\}$, where $\mathcal{S}(f)$ is the set of all meromorphic functions $\alpha(z)$ such that $T(r, \alpha) = S(r, f)$. It is obvious that the conditions of sharing values in Theorem 1.1 could be rewritten as "$E(a, f) = E(a, \Delta_{c=1}^n f)$, $E(b, f) = E(b, \Delta_{c=1}^n f)$ and $E(c, f) = E(c, \Delta_{c=1}^n f)$". So, a natural question is: could those conditions of sharing values be weaken?

In this paper, we give an affirmative answer to the above question and prove a more general result compared to Theorem 1.1 by using a simple method which is very different to the proof of Theorem 1.1. That is

Theorem 1.2 *Let f be a transcendental meromorphic function of hyper order $\rho_2(f) < 1$, and let $c \in \mathbf{C}\backslash\{0\}$ such that $\Delta_c^n f(z) \not\equiv 0$. If*

$$E(1,\, f) = E(1,\, \Delta_c^n f), \quad E(0,\, f) \subset E(0,\, \Delta_c^n f), \quad E(\infty,\, f) \supset E(\infty,\, \Delta_c^n f),$$

then $\Delta_c^n f(z) \equiv f(z)$.

Here, the hyper order $\rho_2(f)$ is defined by $\rho_2(f) = \limsup\limits_{r\to\infty} \dfrac{\log^+ \log^+ T(r,\, f)}{\log r}$ as usual. Clearly, our result improves Theorem 1.1.

2 Some Lemmas

To prove our result, we need the following auxiliary results.

Lemma 2.1[3],[6] *Let $f(z)$ be a nonconstant meromorphic function of hyper order $\rho_2(f) < 1$ and $c \in \mathbf{C}\backslash\{0\}$, n be a positive integer. Then for any $a \in \mathbf{C}$, we have*

$$m\left(r,\ \frac{\Delta_c^n f(z)}{f(z) - a}\right) = S(r,\, f).$$