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Optimal Defined Contribution Pension Management with Salary and Risky Assets Following Jump Diffusion Processes

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Abstract. The paper considers an optimal asset allocation problem for a defined contribution pension plan during the accumulation phase. The salary follows a stochastic process, which combines a compound Poisson jump with Brownian uncertainty. The plan aims to minimise the quadratic loss function over finite time horizon by investing in the market of risky assets and bank account. The risky assets are subjected to Poisson jump and Brownian motion. The closed-form optimal investment decision is derived from the corresponding Hamilton-Jacobi-Bellman equation.

AMS subject classifications: 97M30, 93E20

Key words: Compound Poisson process, defined contribution pension plan, stochastic optimal control, dynamic programming approach, Hamilton-Jacobi-Bellman equation.

1. Introduction

Pension is an important financial instrument for individuals to reallocate their wealth from working life to retirement. Generally, there are two kinds of pension arrangements — viz. defined contribution (DC) and defined benefit (DB) plans. In the former case, the contributions are fixed in advance by the pension sponsor and the benefits depend on the investment earnings, so that the majority of risks are borne by the individual itself. In the other pension plan, the benefits are fixed in the contract, while the contributions are designed by the sponsor to keep the fund in balance. Thus the sponsor bears the majority of risks and the individual does not experience any losses.

Generally, a pension plan consists of accumulation and decumulation phases. In the first phase, also called the contribution phase, the pensioner pays contributions to the pension

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trustee during the employment period. In the other one, a pension annuity or a lump sum is received and can be converted to a whole life assurance with a death benefit.

There are a variety of works concentrating on the optimal investment and the management of DC pension scheme. For instance, Haberman and Vigna [12] used a dynamic programming approach to derive a formula for the optimal investment allocation in the DC pension scheme whose funds are invested in an n asset market. For general multi-asset financial markets with stochastic investment opportunities and stochastic contributions, Menoncin and Vigna [15] solved a mean-variance optimisation problem in the accumulation phase of DC pension schemes. Under the requirement of inflation protected guarantee, Tang *et al.* [19] obtained an optimal asset allocation decision for economic environment with risks arising from real and nominal interest rates. Other optimal management and investment problems for DC pension funds have been recently studied by Wang *et al.* [21] and Josa-Fombellida *et al.* [8].

In order to model the dynamics of risky assets, a geometric Brownian motion is widely used. However, it does not properly match the market prices, so that the jump diffusion model seems to be more appropriate. Considering the optimal consumption and portfolio rules, Merton [16] studied the Poisson jumps in a dynamic portfolio problem. Ngwira and Gerrard [17] investigated DB pension management problems and developed an optimal contribution and investment strategy incorporating jumps into the risky asset price process. Delong *et al.* [5] and Liang *et al.* [14] considered mean-variance problems under the assumption that the dynamics of the stock price is driven by a Lévy noise. On the other hand, the jump diffusion stock price model for the DC pension management remains little studied. In particular, Sun *et al.* [18] analysed the precommitment and equilibrium investment strategies under the assumption that the stock prices follow a jump diffusion process.

However, using the approach of Sun *et al.* [18], one can show that the investment strategy is only optimal at time 0, and the pension trustee is assumed to be precommitted to the target determined at the initial time. This problem motivated us to use the dynamic programming approach to consider a closed-form investment strategy for DC type pension plans. The stock price is stochastic and driven by a Brownian motion and a compound Poisson jump. In contrast to [18], investment strategy in our work is optimal not only at the initial time but also in what follows.

It is worth well to use the jump diffusion processes to study the salary dynamics, since salary may not raise gradually and continuously. It can have a positive jump when the employee is promoted from a lower to a higher position or if another company proposes a higher salary. In fact, such models are already used in the salary studies — e.g. assuming that salary follows a jump diffusion process, Bian *et al.* [1] developed an optimal retirement strategy of a DB type pension plan. The model is also employed by Calvo-Garrido and Vázquez [4] in pricing pension plans.

We also note that in DB pension scheme, the benefits are partially or totally determined by salaries. In particular, a shock in salary can produce shocks in the evolution of benefits. Therefore, Josa-Fombellida and Rincón-Zapatero [9] consider jumps in the evolution of benefits in a DB pension plan. This is another reason for using the jump diffusion processes