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Quenching Time Estimates for Semilinear Parabolic Equations Controlled by Two Absorption Sources in Control System

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Abstract. This paper deals with the quenching solution of the initial boundary value problem for aclass of semilinear reaction-diffusion equation controlled by two absorption sources in control system and estimate upper bound and lower bound of the quenching time. We point that the number of absorption sources influences the time of quenching phenomenon.The solution can solve some boundary value problem in control system.

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1 Introduction

The purpose of the present paper is to consider the quenching phenomenon for the initial boundary value problem (IBVP) of semilinear reaction-diffusion equation

$$u_t(x,t)-\Delta u(x,t)=(b-u(x,t))^{-p}+(b-u(x,t))^{-q} \quad \text{in } \Omega\times(0,T), \tag{1.1}$$
$$u(x,t)=0 \quad \text{on } \partial\Omega\times(0,T), \tag{1.2}$$

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$$u(x,0)=u_0(x)\geq 0 \qquad \text{in } \Omega, \tag{1.3}$$

where $2<p<q$, $b=\text{const}>0$, $\Omega\subset\mathbb{R}^N$ is a bounded domain, $\partial\Omega$ is its smooth boundary, and $u_0(x)$ is the nonnegative initial data in $C^1(\bar{\Omega})$ and $\sup\limits_{x\in\Omega} u_0(x)<b$. We define $(0,T)$ to be the maximal existence time interval of the solution u of (1.1)-(1.3) throughout the whole paper. The solution $u(x,t)$ of (1.1)-(1.3) has the following properties: $u(x,t)$ has twice continuous derivate in $x\in\Omega$ and once in $u(x,t)<b$ for all $t\in(0,T)$. Problem (1.1)-(1.3) represents an elastic membrane inside an idealized electrostatically actuated MEMS.

Definition 1.1. *If $T=+\infty$, we say problem* (1.1)-(1.3) *admits a global solution. If $T<\infty$ and the solution $u(x,t)$ of problem* (1.1)-(1.3) *has a singularity*

$$\lim_{t\to T}\sup_{x\in\Omega} u(x,t)=b,$$

then the solution $u(x,t)$ is the so-called quenching solution of problem (1.1)-(1.3), *T is the quenching time.*

In 1975, Kwawarada [1] investigated the quenching phenomena firstly, formed the basis for further investigation by various authors [2]- [11]. Particularly, Boni and Bernard [7] studied a class of parabolic model with a single absorption source

$$u_t=Lu+r(x)(b-u)^{-p}, \qquad (x,t)\in\Omega\times(0,T), \tag{1.4}$$

$$u=0, \qquad (x,t)\in\partial\Omega\times(0,T), \tag{1.5}$$

$$u(x,0)=u_0(x)\geqslant 0, \qquad x\in\Omega. \tag{1.6}$$

Further, they obtained the quenching phenomena of problem (1.4)-(1.6) and estimated the quenching time. Also, they clearly demonstrated that the absorption source term has an pronounce affect on the quenching phenomenon for the nonlinear reaction diffusion equation. Xu [9] investigated initial boundary value problem (1.4)-(1.6) for nonlinear parabolic differential equations with several combined nonlinearities and carries out numerical experiments. Selcuk [10] and Ozalp [11] showed quenching phenomenon occurs on the singular boundary conditions. The present paper focuses on the solution of the same type of equation with two absorption sources, which are both positive. We change the exponents of both two absorption sources such that the two terms have a large enough gaps in the sense of growth order, in order to reveal and compare the importance of the two factors acting on the behavior of the quenching phenomena, which are the number of the absorption source terms and the exponents of these terms. The results obtained in the preset paper suggests the dominant influence of the exponents of the absorption terms comparing the number of them, which is not only different from the classical heat equation with nonlinear power-type external force [12, 13], but also different from the nonlinearities and their corresponding behaviours and affects in other models [14–30].