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# Lump Solutions of the Modified Kadomtsev-Petviashvili-I Equation

Xuelin Yong[1,*], Yuning Chen[1], Yehui Huang[1,3] and Wen-Xiu Ma[2,3,4,5,6]

[1] *School of Mathematical Sciences and Physics, North China Electric Power University, Beijing 102206, P.R. China.*
[2] *Department of Mathematics, Zhejiang Normal University, Jinhua 321004, Zhejiang, P.R. China.*
[3] *Department of Mathematics and Statistics, University of South Florida, Tampa, FL 33620, USA.*
[4] *School of Mathematics, South China University of Technology, Guangzhou 510640, P.R. China.*
[5] *College of Mathematics and Physics, Shanghai University of Electric Power, Shanghai 200090, P.R. China.*
[6] *International Institute for Symmetry Analysis and Mathematical Modelling, Department of Mathematical Sciences, North-West University, Mafikeng Campus, Private Bag X2046, Mmabatho 2735, South Africa.*

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**Abstract.** The modified Kadomtsev-Petviashvili-I equation is studied by the Hirota bilinear method. Certain lump solutions of this equation are found via the ansatz technique. Rational solutions presented include plane bounded lumps, which do not decay in all directions in the space.

**AMS subject classifications**: 35Q51, 35Q53, 37K40

**Key words**: Lump, symbolic computation, Hirota bilinear method.

## 1. Introduction

As a $(2+1)$-dimensional integrable generalisation of the modified Korteweg-de Vries equation, the modified Kadomtsev-Petviashvili (mKP) equation

$$V_t + V_{xxx} - \frac{3}{2}V^2 V_x + 3\sigma^2 \partial_x^{-1} V_{yy} - 3\sigma V_x \partial_x^{-1} V_y = 0, \tag{1.1}$$

*Corresponding author. *Email addresses:* `mawx@cas.usf.edu` (W.-X. Ma), `yongxuelin@126.com` (X. Yong), `ncepumath@sohu.com` (Y. Chen), `yhhuang@ncepu.edu.cn` (Y. Huang)

 

where $\sigma^2 = \pm 1$, was introduced within the framework of the gauge-invariant description of the KP equation in [13]. In [11], it appeared as the first member of the first modified KP hierarchy. By introducing a new dependent variable defined as $V = \sigma U$, Eq. (1.1) becomes

$$U_t + U_{xxx} - 3\sigma^2 \left( \frac{1}{2} U^2 U_x - \partial_x^{-1} U_{yy} + U_x \partial_x^{-1} U_y \right) = 0, \tag{1.2}$$

which is classified as the modified Kadomtsev-Petviashvili-I (mKPI) equation when $\sigma = i$ and the modified Kadomtsev-Petviashvili-II (mKPII) equation when $\sigma = 1$ [44]. Both mKPI and mKPII equations are physically significant nonlinear evolution equations. They can be solved by the inverse scattering transform (IST) method, the Darboux transform method, the $\bar{\partial}$-dressing method, the Hirota bilinear method — cf. Refs. [1,2,6,14–17,25,29,45].

Lump solutions are a kind of analytic rational function solutions, localised in all directions in the space. General rational function solutions of the Korteweg-de Vries equation, the Boussinesq equation and the Toda lattice equation have been studied by using Wronskian and Casoratian determinant [5,22–24]. Special lumps also appear as solutions of KPI, BKP, Davey-Stewartson-II and Ishimori-I equations [3,7,10,12,18,28,31,36–39]. Although for mKP and KP equations the $2+1$-dimensional Miura transformation exists, it does not convert real solutions of mKPI and KPI equations into each other [15]. Therefore, it would interesting to find an efficient way for finding real rational solutions of the mKPI equation.

Based on the Hirota bilinear method [9], one of the authors (Ma) proposed a direct method for determining of positive quadratic function solutions to the $(2+1)$-dimensional bilinear KPI equation [20] and general Hirota bilinear equations [21]. The same approach applies to many other equations [8, 19, 26, 30, 32–35, 41, 42, 46–48]. The method has been also recently used to characterise the lump solutions of the KPI equation with a self-consistent source [43].

In this work, we employ Maple symbolic computation, to present two general classes of lump solutions of the mKPI equation (1.2). This equation (1.2) has a Hirota bilinear form and we use special ansätz to find real rational solutions. The solutions obtained contain free parameters, a special choice of which covers lump solutions generated from the IST. In addition, they also generate plane bounded lumps, which do not decay in all directions in the space. Finally, a few concluding remarks are given at the end of the paper.

## 2. Lump Solutions of mKPI Equation

Using the variable transformation $U = 2i(\ln(G/F))_x$, one can write the mKPI equation (1.2) as

$$\begin{aligned} &\left(D_t + D_x^3 + 3iD_x D_y\right) G \cdot F = 0, \\ &\left(D_x^2 - iD_y\right) G \cdot F = 0 \end{aligned} \tag{2.1}$$

with the $D$-operators

$$D_t^m D_x^n G \cdot F = \left( \frac{\partial}{\partial t} - \frac{\partial}{\partial t'} \right)^m \left( \frac{\partial}{\partial x} - \frac{\partial}{\partial x'} \right)^n G(x,t) F(x',t') \bigg|_{x'=x, t'=t}.$$