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An Efficient Numerical Method for the Valuation of American Better-of Options Based on the Front-Fixing Transform and the Far Field Truncation

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Abstract. In this paper, an efficient numerical method is proposed for the valuation of American better-of options based on the Black-Scholes model. Because of the existence of the optimal exercise boundary, the American better-of option satisfies a two dimensional parabolic linear complementarity problem on an unbounded domain. We first transform it into a one dimensional free boundary problem by a standard change of variables. And then the front-fixing transformation and the far field truncation are used to deal with the free boundary and the unbounded domain in succession, which yields a parabolic problem with unknown coefficient (free boundary) on a bounded regular domain. Furthermore, a finite element method is applied to discretize the resulting continuous system. The stability of the semi-discrete solution is also established. Meanwhile, Newton's method is used to solve the discretized system to obtain the free boundary and the option value simultaneously. The nonnegativity of the iteration solutions is also proved. Finally, numerical simulations are carried out to test the performance of the proposed method.

AMS subject classifications: 35A35, 90A09, 65K10, 65M12, 65M60

Key words: American better-of option, far field truncation, front-fixing transformation, finite element method.

1 Introduction

Option is a contract that can be exercised on the maturity date or at any time before the maturity date. The former one is called the European-style option, which usually has closed form solution [14]. The latter one is named the American-style option, which has no closed form solution because of the existence of an early exercise right. Due to

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the flexibility in choosing exercise time, American-style options are very popular with investors. There is a great deal of research on the classical one-asset option, we refer to [3, 5, 6, 17, 22, 25] and references therein for the rich literature. With the development of financial market and the change of demand, other kinds of American options, such as basket options, lookback options, and better-of options etc., have emerged. In this paper, we mainly focus on the research of American better-of options. American better-of option is a kind of multi-asset option, with which the owner can choose one or several beneficial assets to exercise the contract, in order to optimize the benefit. Similar to other American-style options, better-of option also has no closed form solution. Thus its research has to rely on numerical approaches.

Based on the Black-Scholes model, the easiest numerical method for the valuation of the classical American options is the binomial method. It is a tree-based method, which was first proposed in [7], and proved to be convergent by Amin and Khanna [1]. However, the computational cost of the tree-based methods increases rapidly with the increase of the number of underlying assets, which makes it impractical. Based on the fact that Black-Scholes model is a parabolic problem, the finite difference method [3,15,24,26] and the finite element method [2, 28] were also proposed to price American options. They all perform well under some special conditions. For other efficient numerical methods, the interested readers could refer to references [12, 18, 20, 23]. Inspired by the traditional American option pricing methods, we will apply the finite element method to deal with better-of options. American better-of options satisfy a two dimensional parabolic linear complementarity problem on an unbounded domain, which can not be solved easily. To solve it efficiently, we must overcome (1) the solution domain (holding domain after transformations) is unbounded; (2) the optimal exercise boundary is an unknown curve. These two aspects will affect the design of an algorithm directly. Moreover, we also need to face with (3) proposing an efficient approach to obtain the option value and the optimal exercise boundary simultaneously.

In order to simplify the pricing model, we first transform it into a one dimensional free boundary (optimal exercise boundary) problem on an unbounded domain by a standard change of variables, and then we deal with the above issues one by one. There are basically two kinds truncation techniques which can be used to handle the unbounded domain. One is the nonlocal boundary condition method(is also called the transparent boundary condition), which was proposed and applied to the valuation of classical American options in 2003 by Han and Wu [11], and improved by M. Ehrhardt and R. E. Mickens [9] in 2008. The other is the far field boundary condition method [17], which was applied to price American options in 2008 by Homels and Yang [13]. Since the former truncation technique can not be easily combined with the finite element method, we will use the latter one to deal with our problem. The optimal exercise boundary corresponding to the free boundary, which is unknown a priori, is a curve along the temporal direction. For this kind of moving boundary problem, the efficient techniques include the Laudau's transformation [21] and the front-fixing transformation [8, 13, 23]. These two techniques have similar effects, all could transform the free boundary problem into