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SOME MATHEMATICAL MODELS AND MATHEMATICAL ANALYSIS ABOUT RAYLEIGH-TAYLOR INSTABILITY*

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Abstract

In this paper we will review some mathematical models and mathematical analysis about Rayleigh-Taylor instability.

Keywords mathematical models; mathematical analysis; Rayleigh-Taylor instability

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1 Introduction

The instability of the interface between two different densities of fluid under the action of gravity or inertial force, as early as 1950, was clearly pointed out by G.L. Taylor, and is often named after him, actually earlier than him, L. Rayleigh in 1900 and S.H. Lamb in 1932 also talked about this problem in some sense, people sometimes call Rayleigh-Taylor or Rayleigh-Lamb-Taylor instability. This interfacial instability phenomenon can be found not only in astrophysics, but also in laster fusion and high-speed collision. It is very important even for hydraulic machinery and various engines. The linear development stage of interface instability is relatively clear. However, there are still many problems in nonlinear development that need to be recognized. The relevant research has very important practical and theoretical value.

In the mathematical analysis theory, since 2003, there have been some breakthrough works on the RT instability of compressible fluids [1,2], free boundary prob-

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lems [3] and MHD fluids [4]. Of course, these theoretical results are still far away form the actual physical mechanics. There is still a big gap in the problem.

2 Some RT Instability Mathematical Models

2.1 Double infinite fluid Taylor instability

We consider the two-layer infinite fluid shown in Figure 2.1, where each of densities ρ_1 and ρ_2 occupies a half plane of $y > 0$ and $y < 0$, and gravity $\vec{g}$ is parallel to y axis, pointing to its negative direction, that is $\vec{g} = -g\vec{j}$.

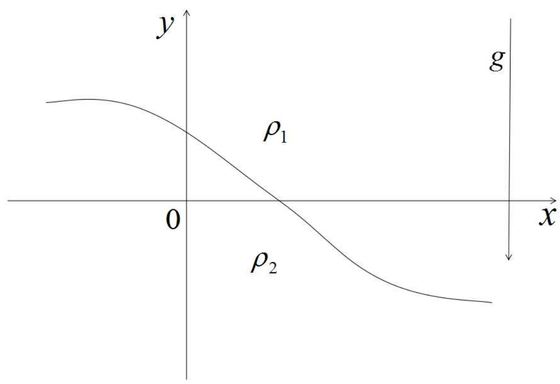

Figure 2.1

The deviation of the initial position of the interface from $y = 0$ is a small amount, that is $y = \varepsilon \cos kx$.

Assume that the two-layer fluid is in a static state at the initial moment, and thus is initially non-rotating. For an ideal incompressible fluid, the fluid remains free-curl in the subsequent movement. Thus, the velocity potential Φ_i $(i = 1, 2)$ can be introduced corresponding to the upper layer and lower layer of fluid. Then under two-dimensional conditions, Φ_i $(i = 1, 2)$ satisfies the Laplace equation

$$\Phi_{ixx} + \Phi_{iyy} = 0, \quad i = 1, 2. \tag{2.1}$$

If we suppose that the interface position during the movement is $y = \eta(x, t)$. And its derivative is the first-order small amount of ε, then

$$F(r, t) = y - \eta(x, t) = 0. \tag{2.2}$$

Therefore,

$$\begin{aligned} &\frac{\partial F}{\partial t} = -\eta_t \sim \varepsilon, \\ &\frac{\partial F}{\partial \eta} = 1, \quad \frac{\partial F}{\partial x} = -\eta_x \sim \varepsilon, \\ &\nabla F = -\eta_x \vec{i} + \vec{j}. \end{aligned} \tag{2.3}$$