Commun. Math. Res.
doi: 10.4208/cmr.2020-0034

Vol. **37**, No. 2, pp. 180-208
May 2021

A Level Set Representation Method for N-Dimensional Convex Shape and Applications

Lingfeng Li[1,2], Shousheng Luo[3,*], Xue-Cheng Tai[1] and Jiang Yang[2]

[1] *Department of Mathematics of Hong Kong Baptist University, Hong Kong, SAR.*
[2] *Department of Mathematics of Southern University of Science and Technology, Shenzhen, China.*
[3] *School of Mathematics and Statistics, Henan Engineering Research Center for Artificial Intelligence Theory and Algorithms, Henan University, Kaifeng, China.*

Received 19 September 2020; Accepted 27 November 2020

Abstract. In this work, we present a new method for convex shape representation, which is regardless of the dimension of the concerned objects, using level-set approaches. To the best of our knowledge, the proposed prior is the first one which can work for high dimensional objects. Convexity prior is very useful for object completion in computer vision. It is a very challenging task to represent high dimensional convex objects. In this paper, we first prove that the convexity of the considered object is equivalent to the convexity of the associated signed distance function. Then, the second order condition of convex functions is used to characterize the shape convexity equivalently. We apply this new method to two applications: object segmentation with convexity prior and convex hull problem (especially with outliers). For both applications, the involved problems can be written as a general optimization problem with three constraints. An algorithm based on the alternating direction method of multipliers is presented for the optimization problem. Numerical experiments are conducted to verify the effectiveness of the proposed representation method and algorithm.

*Corresponding author. *Email address:* sluo@henu.edu.cn (S. Luo)

AMS subject classifications: 65D18, 65K10

Key words: Convex shape prior, level-set method, image segmentation, convex hull, ADMM.

1 Introduction

In the tasks of computer vision, especially image segmentation, shape priors are very useful information to improve output results when the objects of interest are partially occluded or suffered from strong noises, intensity bias and artifacts. Therefore, various shape priors are investigated in the literature [5, 14, 20, 33]. In [8, 20], the authors combined shape priors with the snakes model [2] using a statistical approach and a variational approach, respectively. Later, based on the Chan-Vese model [5], a new variational model, which uses a labeling function to deal with the shape prior, was proposed in [11]. A modification of this method was presented in [4] to handle the scaling and rotation of the prior shape. All the priors used in these papers are usually learned or obtained from some given image sets specifically.

Recently, generic and abstract shape priors have attracted more and more attention, such as connectivity [36], star shape [34, 39], hedgehog [18] and convexity [15, 33]. Among them, the convexity prior is one of the most important priors. Firstly, many objects in natural and biomedical images are convex, such as balls, buildings, and some organs [30]. Secondly, convexity also plays a very important role in many computer vision tasks, like human vision completion [23]. Several methods for convexity prior representation and its applications were discussed in the literature [14, 33, 38]. However, these methods often work for 2-dimensional convex objects only and may have relatively high computational costs. In this paper, we will present a new method for convexity shape representations which is suitable for high dimensional objects.

Most of the existing methods for convex shape representation can be divided into two groups: discrete approaches and continuous approaches. For the first class, there are several methods in the literature. In [31], the authors first introduced a generalized ordering constraint for convex object representation. To achieve the convexity of objects, one needs to explicitly model the boundaries of objects. Later, an image segmentation model with the convexity prior was presented in [14]. This method is based on the convexity definition and the key idea is penalizing all 1-0-1 configurations on all straight lines where 1 (resp. 0) represents the associated pixel inside (resp. outside) the considered object. This