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Homoclinic Orbits of a Quadratic Isochronous System by the Perturbation-incremental Method*

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Abstract In this paper, the perturbation-incremental method is presented for the analysis of a quadratic isochronous system. This method combines the remarkable characteristics of the perturbation method and the incremental method. The first step is the perturbation method. Assume that the parameter λ is small, i.e. $\lambda \approx 0$, the initial expression of the homoclinic orbit is obtained. The second step is the parameter incremental method. By extending the solution corresponding to small parameters to large parameters, we can get the analytical-expressions of homoclinic orbits.

Keywords Perturbation-incremental method, Homoclinic orbits, Quadratic isochronous system.

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1. Introduction

Considering the following plane system [7, 33]

$$\begin{cases} \dfrac{dx}{dt} = y + \lambda f(x,y), \\ \\ \dfrac{dy}{dt} = -g(x) + \lambda h(x,y), \end{cases} \tag{1.1}$$

where f, g and h are arbitrary nonlinear functions of their arguments, λ is a real parameter of arbitrary magnitude. If $f(0,0)$=$g(0)$=$h(0,0)$=0, the origin is a singular point. When $0 < \lambda < \overline{\lambda}$, equation (1.1) has a limit cycle around the origin. On the other hand, if λ is specified, then the analytical-expressions of limit cycles and

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homoclinic (heteroclinic) orbits will be calculated as given before in [5,14,31]. In practice, many quantitative methods are used to solve the analytical-expressions of limit cycles and homoclinic orbits such as Incremental Harmonic Balance method (IHB) [18,19], Elliptic-Perturbation method (EP) [6,10], Lindst-Poincaré method (LP) [11,27], perturbation iteration method [4].

In recent years, many scholars have begun to pay attention to limit cycle bifurcations of isochronous systems [2,22,23,26,29]. Loud [24] has divided quadratic polynomial differential systems having an isochronous center into four classes S_1, S_2, S_3 and S_4. Yang [35] obtained the upper bounds of the number of limit cycles bifurcating from the period annuli of quadratic isochronous systems (S_1 and S_2) by using the Picard-Fuchs equation. Li [20] investigated the number of limit cycles which bifurcate from the period annulus of a class of quadratic isochronous system (S_3).

In this article, the perturbation-incremental method [5,31] is given for the calculation of homoclinic orbits of quadratic isochronous differential systems [20,35]. This method is especially suitable for some systems with parameters. When parameters of systems are small, the perturbation method is used to give the zero-order perturbation solution the analytical-expressions of homoclinic orbits. When parameters are gradually increasing, the parameter incremental method and the iterative method are used to extend the solution corresponding to small parameters to large parameters, and the analytical-expressions of homoclinic orbits satisfying the required accuracy are obtained.

Perturbation-incremental method is a new method combining the semi-analytical method with the numerical method, and has been developed for a long time. Xu et al. [32] applied the perturbation-incremental method to the calculation of limit cycles and homoclinic (heteroclinic) orbits of strong nonlinear oscillators in electrical engineering. Huang et al. [16,17] used the perturbation-incremental method to discuss the limit cycles, homoclinic orbits and the quantitative analysis of parameters bifurcation of Bogdanov-Takens system. Chen et al. [8] and Lin [21] used the perturbation-incremental method to study the approximate solution of semi-stable limit cycles of Liénard equation, and as well calculation of multiple limit cycles with their bifurcation values. In the following years, the perturbation-incremental method has been widely used in the calculation of periodic solutions of nonlinear systems of delay differential equations [3,12,30], bifurcation of impulsive systems [28], calculation of limit cycles, homoclinic (heteroclinic) orbits and bifurcation of general dynamical systems [1,9,13,15,25].

Next, we will describe the main contents of this method and give an example.

2. Perturbation-incremental method

The common nonlinear oscillators systems and quadratic systems can be reduced to the form of (1.1). We introduce a nonlinear time transformation of the form

$$\frac{d\varphi}{dt} = \Phi(\varphi), \quad \Phi(\varphi + 2\pi) = \Phi(\varphi), \tag{2.1}$$

where φ is the new time. In the φ domain, equation (1.1) has the form

$$\Phi\frac{dx}{d\varphi} = y + \lambda f(x, y), \quad \Phi\frac{dy}{d\varphi} = -g(x) + \lambda h(x, y). \tag{2.2}$$