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A Priori and a Posteriori Error Analysis of the Discontinuous Galerkin Methods for Reissner-Mindlin Plates

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Abstract. In this paper, we apply an a posteriori error control theory that we develop in a very recent paper to three families of the discontinuous Galerkin methods for the Reissner-Mindlin plate problem. We derive robust a posteriori error estimators for them and prove their reliability and efficiency.

AMS subject classifications: 65N30, 65N15, 35J25

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1 Introduction

This paper will consider a posteriori error analysis of finite element methods for the Reissner-Mindlin plate problem: given $g\in L^2(\Omega)$ find

$$(w,\boldsymbol{\phi})\in W\times\boldsymbol{\Theta}:=H_0^1(\Omega)\times H_0^1(\Omega)^2,$$

with

$$a(\boldsymbol{\phi},\boldsymbol{\psi})+(\gamma,\nabla v-\boldsymbol{\psi})_{L^2(\Omega)}=(g,v)_{L^2(\Omega)},\quad \text{for all } (v,\boldsymbol{\psi})\in W\times\boldsymbol{\Theta}, \tag{1.1}$$

and the shear force

$$\gamma=\lambda t^{-2}(\nabla w-\boldsymbol{\phi}). \tag{1.2}$$

Here and throughout this paper, t denotes the plate thickness, $\lambda{=}Ek/2(1+\nu)$ the shear modulus, E the Young modulus, ν the Poisson ratio, and κ the shear correction factor. Given $\boldsymbol{\phi}\in\boldsymbol{\Theta}$, the linear Green strain $\varepsilon(\boldsymbol{\phi}){=}1/2[\nabla\boldsymbol{\phi}+\nabla\boldsymbol{\phi}^T]$ is the symmetric

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part of gradient field $\nabla\boldsymbol{\phi}$. For all 2×2 symmetric matrices the linear operator $\mathcal{C}$ is defined by

$$\mathcal{C}\boldsymbol{\tau} := \frac{E}{12(1-\nu^2)}\big[(1-\nu)\boldsymbol{\tau} + \nu\,\mathrm{tr}(\boldsymbol{\tau})I\big].$$

The bilinear form $a(\cdot,\cdot)$ models the linear elastic energy defined as

$$a(\boldsymbol{\phi},\boldsymbol{\psi}) = (\mathcal{C}\varepsilon(\boldsymbol{\phi}),\varepsilon(\boldsymbol{\psi}))_{L^2(\Omega)}, \quad \text{for any } \boldsymbol{\phi},\ \boldsymbol{\psi}\in\boldsymbol{\Theta}, \tag{1.3}$$

which gives rise to the norm

$$\|\boldsymbol{\psi}\|_{\mathcal{C}}^2 := a(\boldsymbol{\psi},\boldsymbol{\psi}), \quad \text{for any } \boldsymbol{\psi}\in\boldsymbol{\Theta}, \tag{1.4}$$

while $\|\cdot\|_{\mathcal{C}_h}$ denotes the broken version with the piecewise defined operator ε_h taking the place of ε, and $(\cdot,\cdot)_{L^2(\Omega)}$ the L^2 scalar product.

This plate theory has become a popular plate bending model in the engineering community due to its simplicity and effectiveness. However, a direct finite element approximation usually yields poor numerical results, i.e., they are too small compared with the continuous solutions. Such a phenomenon is usually referred to as *shear locking*. To weaken or even overcome the locking, many methods have been proposed, most of them can be regarded as reduction integration methods. Very recently, three class of the discontinuous Galerkin methods are used to discretize the Reissner-Mindlin plate problems [1,2]. The aim of this paper is to provide a robust a prior and a posteriori error analysis for these methods.

2 Notation and preliminary results

We use the standard differential operators:

$$\nabla r = \Big(\frac{\partial r}{\partial x},\frac{\partial r}{\partial y}\Big), \qquad \mathrm{curl}\, p = \Big(\frac{\partial p}{\partial y},-\frac{\partial p}{\partial x}\Big).$$

Given any $2D$ vector function

$$\boldsymbol{\psi} = (\psi_1,\psi_2),$$

its divergence reads

$$\mathrm{div}\,\boldsymbol{\psi} = \frac{\partial\psi_1}{\partial x} + \frac{\partial\psi_2}{\partial y}.$$

With the differential operator

$$\mathrm{rot}\,\boldsymbol{\psi} = \frac{\partial\psi_2}{\partial x} - \frac{\partial\psi_1}{\partial y},$$

for a vector function $\boldsymbol{\psi}$=(ψ_1,ψ_2), the space $H_0(\mathrm{rot},\Omega)$ is defined as

$$H_0(\mathrm{rot},\Omega) := \big\{\boldsymbol{v}\in L^2(\Omega)^2,\ \ \mathrm{rot}\,\boldsymbol{v}\in L^2(\Omega) \text{ and } \boldsymbol{v}\cdot\boldsymbol{\tau} = 0 \text{ on } \partial\Omega\big\},$$