

A Consumption Behavior Model with Advertising And Word-of-Mouth Effect

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Abstract It is widely-held belief that people's consumption behavior is partly determined by advertising and word-of-mouth effect especially in monopolistic competitive market. Owing to this, we propose a mathematical model to interpret consumer behavior under the advertising and word-of-mouth effects, which is divided into continuous and discrete types for dynamic behavior analysis. Our research indicates the continuous model undergoes fold bifurcation, Hopf bifurcation, and degenerate fold-Hopf bifurcation; the discrete model undergoes flip bifurcation and Neimark-Sacker bifurcation. Moreover, bifurcation diagrams are given by using MATLAB to illustrate the model. Based on the theory of Hopf bifurcation or Flip bifurcation, the system undergoes supercritical Hopf bifurcation or Flip bifurcation under certain conditions. This lead to both advertising impacts periodically on consumer behavior, and a short-term reduction cause no effect the public's recognition of the brand, which can guide precision advertising investment.

Keywords The consumption behaviour model, Hopf bifurcation, fold bifurcation, Flip bifurcation, Neimark-Sacker bifurcation.

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1. Introduction

As the main way for consumers to understand product information, commercial advertising spreads new consumption concepts, supplies new consumption methods, and guides consumer demand. Advertisement also play an important role in four different market conditions, namely perfect competition, monopolistic competition, oligopoly and monopoly. Under the oligopoly competition market, for example, the international company Coca-Cola invest huge amounts of money in a large number of advertisements, creating fashion slogans all the time, planning creative advertisements, and leads young people's demand for Coca-Cola. As a corporation in monopolistic competition market, Apple company promoted the Spring Festival brand advertisement in China since 2015. We note in Figure 1 (Data are from Apple quarterly earnings *) that revenue in Greater China have increased significantly in

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2015 Q1, 2016 Q1, 2017 Q1 and 2018 Q1 (Q1 indicates the first quarter). This conveys that the effective advertising is able to raise the maximum price consumers are willing to pay, product sales and eventually to boost profits.



Figure 1. Apple's quarterly earnings report for Greater China.

Advertising is vital in our daily life. It mainly determines the impact on our thinking as well as on the attitude towards different and new products. An excellent and creative advertisement can disseminate, arouse people's desire to buy and thus realize the commodity value. While the impacts of advertising and sales promotion in revenue are often used interchangeably, advertising and sales promotion designate different parts in enterprise marketing strategy. Advertising has to do with building brand recognition and taking measures in order to secure long-term profitability, whereas sales promotions are short-term strategies infusing immediate revenue into a business by directly affecting the price of goods or services.

In one social science study [1], the negative and positive effect of word of mouth marketing on consumer purchasing behavior have been examined. The study is based on primary data collected from numerous households and university students from the area of Rawalpindi and Islamabad cities of Pakistan, the results have revealed that consumers tend to rely on word of mouth for the purchase of not only everyday items, but long-term goods as well. The person who have a great influence on the decision of consumers are close family, friends and acquaintances. Moreover, the research imputes the failure of revenue-poor companies mostly to adverse word of mouth. Viral Marketing is gaining rapid popularity among consumers as it is depicted in the results. Finally, a negative word of mouth travels faster and more widely spread than positive comments.

In view of the fact that advertising and word-of-mouth effects are dynamic economic phenomena, many studies have used the Innovation diffusion theory because of its great economic importance since its appearance in the previous century sixties. Numerous mathematical models have been established to market new products. Sales Promotions were widely discussed in many studies from both mathematical and economic standpoints. Many of these models aim to explain the spread of a new