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Stabilization for a Fourth Order Nonlinear Schrödinger Equation in Domains with Moving Boundaries

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Abstract. In the present paper we study the well-posedness using the Galerkin method and the stabilization considering multiplier techniques for a fourth-order nonlinear Schrödinger equation in domains with moving boundaries. We consider two situations for the stabilization: the conservative case and the dissipative case.

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1 Introduction

In this work we study the existence of the weak solution as well as the asymptotic behaviour of the fourth-order nonlinear Schrödinger equation in a bounded domain with moving boundary. Indeed, we consider the following initial value problem

$$u_\tau - i u_{\xi\xi\xi\xi} + |u|^2 u + \eta u = 0 \qquad \text{in } Q_\tau, \tag{1.1a}$$
$$u(\xi,0) = u_0(\xi) \qquad \text{in } I_\tau, \tag{1.1b}$$
$$u(0,\tau) = u(1,\tau) = 0 \qquad \text{on } \Sigma_\tau, \tag{1.1c}$$

where η is real positive constant and $i=\sqrt{-1}$, which occurs over different point views, for instance, solitary waves [1,2], in the study of deep water dynamics [3], vortex filaments [4]. In general, The physical models (1.1a) occur in various areas of physics, including nonlinear optics, plasma physics, superconductivity and quantum mechanics (see the

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book [5] for a survey on this topics and references therein). By Q_τ we are denoting the time dependent domain of $\mathbb{R}^2$ defined by

$$Q_\tau = \{(\xi,\tau) \in \mathbb{R}^2 : \xi \in I_\tau, \quad 0 < \tau < T\} \tag{1.2}$$

and I_τ is the time dependent interval given by

$$I_\tau = \{\xi \in \mathbb{R} : \alpha(\tau) < \xi < \beta(\tau)\}, \tag{1.3}$$

where $\alpha(\cdot)$, $\beta(\cdot) \in C^2[0,+\infty)$ are such that

$$0 < \alpha_0 \leq \beta(\tau) - \alpha(\tau) \leq \beta_0 < +\infty, \quad \forall \tau \geq 0. \tag{1.4}$$

The lateral boundary Σ_τ of Q_τ is given by

$$\Sigma_\tau = \bigcup_{0<\tau<T} (\alpha(\tau) \times \{\tau\}) \cup (\beta(\tau) \times \{\tau\}). \tag{1.5}$$

There are many results for the fourth order nonlinear Schrödinger equation (see [1,2,6–8] and references cited therein). Especially, the one dimensional case is well studied. The objectives of this work are twofold. To show the existence and uniqueness of the solution to the moving boundary problem (1.1a) and to transform the problem into another initial boundary value problem defined over a cylindrical domain whose sections are not time-dependent. We can to note that the solvability and stability of several evolutions models using this strategy have previously been considered [9–12].

The main result of this paper are the following theorems

Theorem 1.1. *Let $v_0 \in H_0^2(I_0)$ and suppose that the assumption* (1.3) *holds. Then there exists a unique solution u of the problem* (1.1a) *from the class*

$$u \in L^\infty(0, \infty : H_0^2(I_\tau)), \quad u_\tau \in L^\infty(0, \infty : L^2(I_\tau)). \tag{1.6}$$

Theorem 1.2. *Let $v_0 \in H_0^2(I_0)$ and suppose that the assumption* (1.3) *holds. Then the solution given in Theorem* 1.1 *satisfies*

$$\|u(\xi,\tau)\|_{L^2} \leq \frac{1}{(1+\tau)^{1/2}}, \qquad \forall \tau \geq 0 \quad \text{if } \eta = 0, \tag{1.7}$$

$$\|u(\xi,\tau)\|_{L^2} \leq C\exp(-\eta\tau), \qquad \forall \tau \geq 0 \quad \text{if } \eta \neq 0, \tag{1.8}$$

where C and η are positive constants.

Our paper is organized as follows: In Section 2 we transform the moving boundary domain into the cylindrical one. In Section 3 we obtain the existence and uniqueness of solutions to the model. Finally, in Section 4 we analyze the solution of (1.1a) using suitable multiplier techniques.

Finally, throughout this paper C is a generic constant, not necessarily the same at each occasion (it will change from line to line), which depends in an increasing way on the indicated quantities.