A Simple Low-Degree Optimal Finite Element Scheme for the Elastic Transmission Eigenvalue Problem

Yingxia Xi[1], Xia Ji[2,3,*] and Shuo Zhang[4,5]

[1] *School of Science, Nanjing University of Science and Technology, Nanjing 210094, China.*
[2] *School of Mathematics and Statistics, Beijing Institute of Technology, Beijing 100081, China.*
[3] *Beijing Key Laboratory on MCAACI, Beijing Institute of Technology, Beijing 100081, China.*
[4] *LSEC, ICMSEC, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China.*
[5] *University of Chinese Academy of Sciences, Beijing 100049, China.*

Abstract. The paper presents a finite element scheme for the elastic transmission eigenvalue problem written as a fourth order eigenvalue problem. The scheme uses piecewise cubic polynomials and obtains optimal convergence rate. Compared with other low-degree and nonconforming finite element schemes, the scheme inherits the continuous bilinear form which does not need extra stabilizations and is thus simple to implement.

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1 Introduction

The transmission eigenvalue problem is important in the qualitative reconstruction in the inverse scattering theory of inhomogeneous media. For example, the eigenvalues can be used to obtain estimates for the physical characteristics of the hidden scatterer [6, 32] and have a multitude of applications in inverse problems for target identification and nondestructive testing [9, 17]. Besides, the transmission eigenvalues play a key role in the uniqueness and reconstruction in inverse scattering theory. Moreover, they can be

*Corresponding author. *Email addresses:* `xiyingxia@njust.edu.cn` (Y. Xi), `jixia@bit.edu.cn` (X. Ji), `szhang@lsec.cc.ac.cn` (S. Zhang)

used to design invisibility materials [22]. There are different types of transmission eigenvalue problems, such as the acoustic transmission eigenvalue problem, the electromagnetic transmission eigenvalue problem, and the elastic transmission eigenvalue problem, etc. In this paper, we focus ourselves on the elastic transmission eigenvalue problem, which arises in the inverse problems of the elastic wave [7], and thus can be applied in the investigation of the mechanical and dynamical properties of the earth [35].

Since 2010, effective numerical methods for the acoustic transmission eigenvalues have been developed by many researchers [1,8,10,14,15,19,20,23,24,26,28,31,34,39–41], while there are much fewer works for the electromagnetic transmission eigenvalue problem and the elastic transmission eigenvalue problem [18,21,29,33,38,42]. In this paper, we try to develop effective numerical methods for transmission eigenvalue problem of elastic waves.

The non-self-adjointness and nonlinearity plaguing the numerical study of the elastic transmission eigenvalue problem are compounded by the tensorial structure of the elastic wave equation. There exist only a few numerical studies and the theory is far complete. The first numerical treatment appeared in [21]. The authors proposed an secant iterative method to compute the transmission eigenvalues. The elastic transmission eigenvalue problem is reformulated as solving the roots of a nonlinear function of which the values correspond to the generalized eigenvalues of a series of fourth order self-adjoint eigenvalue problems discretized by H^2-conforming finite element methods. However, only real eigenvalues can be captured. Based on H^2-conforming finite element discretization, another work can be found in [42] where the classical H^2-conforming finite element method and the H^2-conforming spectral element method were proposed. The advantage of this approach is that the convergence is automatically guaranteed. The disadvantage is that it requires C^1 finite elements, which can be quite complicated. In [38], Xi et al. propose an interior penalty discontinuous Galerkin method using C^0 Lagrange elements (C^0IP) for the elastic transmission eigenvalue problem which use less degrees of freedom than C^1 elements and much easier to be implemented. There also existed some mixed methods for this problem [37,43]. This approach only requires C^0 finite elements. However, we need to choose finite element pairs satisfying the stability condition. For the non-conforming finite element, to the authors' acknowledge, there didn't exist any related works. A natural thought is to choose the classical Morley element which uses low order quadratic polynomials. It's easy to implement but hence not efficient for capturing smooth solutions. This paper is inspired to present a high order nonconforming finite element method for the elastic transmission eigenvalue problem. Although the existence of elastic transmission eigenvalues is beyond our concern, we want to remark that there exist only a few studies on the existence of the elasticity transmission eigenvalues [4,5,11,12]. We hope that the numerical results can give some hints on the analysis of the elasticity transmission eigenvalue problem.

In this paper, we study the H^2-nonconforming finite element method B_{h0}^3 for the elastic transmission eigenvalue problem. The method is first introduced in [44] for the biharmonic equation with optimal convergence rate. The method does not correspond to