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Asymptotic Behavior of Solutions to One-Dimensional Compressible Navier-Stokes-Poisson Equations with Large Initial Data

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Abstract. In this paper, we are concerned with the large time behavior of global solutions to the Cauchy problem of one-dimensional compressible Navier-Stokes-Poisson equations with density and/or temperature dependent transport coefficients and large initial data. The initial data are assumed to be without vacuum and mass concentrations, and the same is shown to be hold for the global solution constructed. The proof is based on some detail analysis on uniform positive lower and upper bounds of the specific volume and the absolute temperature.

AMS subject classifications: 35Q30, 35B40, 35A01

Key words: Navier-Stokes-Poisson equations, global solutions with large data, density and/or temperature dependent transport coefficients.

1 Introduction and main results

1.1 The problem and our main results

The compressible Navier-Stokes-Poisson (denote as NSP in the sequel) equations

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which take form of compressible Navier-Stokes equations coupled with self-consistent Poisson equation are often used to simulate the motion of viscous fluid under the influence of self-consistent electrostatic potential force. In this paper, we will consider one-dimensional non-isentropic compressible NSP equations, which can be written in the Lagrange coordinates as

$$v_t - u_x = 0, \tag{1.1a}$$

$$u_t + p(v,\theta)_x = \left(\frac{\mu u_x}{v}\right)_x + \frac{\phi_x}{v}, \tag{1.1b}$$

$$\left(E + \frac{u^2}{2}\right)_t + \big(up(v,\theta)\big)_x = \left(\frac{\mu u u_x}{v}\right)_x + \left(\frac{\kappa\theta_x}{v}\right)_x + \frac{u\phi_x}{v}, \tag{1.1c}$$

$$\left(\frac{\phi_x}{v}\right)_x = 1 - ve^{-\phi}, \quad \lim_{|x|\to\infty}\phi(t,x) = 0. \tag{1.1d}$$

Here $t \geq 0$ and $x \in \mathbb{R}$ are the time and Lagrangian spatial variables. The unknown functions $v(t,x)$, $u(t,x)$, $\theta(t,x)$ and $\phi(t,x)$ stand for the specific volume, the velocity, the absolute temperature, and the self-consistent potential, respectively. p and E are the pressure and internal energy. $\mu > 0$ and $\kappa > 0$ are transport coefficients which are assumed to be smooth functions of the specific volume v and/or the absolute temperature θ. Throughout this paper, we focus on the case that the background doping profile is a positive constant which, without loss of generality, can be normalized to be 1. Moreover, we consider the ideal, polytropic gas

$$p = \frac{R\theta}{v}, \quad E = c_v\theta. \tag{1.2}$$

Here c_v and R are the specific heat at constant volume and the specific gas constant, respectively.

This paper is concerned with the problem on the construction of global smooth nonvacuum solutions $(v(t,x), u(t,x), \theta(t,x), \phi(t,x))$ together with the precise description of their large time behaviors to the Cauchy problem of the NSP system (1.1), (1.2) with prescribed large initial data

$$\big(v(0,x), u(0,x), \theta(0,x)\big) = \big(v_0(x), u_0(x), \theta_0(x)\big), \tag{1.3}$$

which is further assumed to satisfy the following far field conditions:

$$\lim_{|x|\to\infty}\big(v_0(x), u_0(x), \theta_0(x)\big) = \big(v_\pm, u_\pm, \theta_\pm\big). \tag{1.4}$$

This paper is concentrated on the case when the far fields $(v_\pm, u_\pm, \theta_\pm)$ of the initial data $(v_0(x), u_0(x), \theta_0(x))$ are the same, i.e. $(v_-, u_-, \theta_-) = (v_+, u_+, \theta_+)$, and without loss of generality, one can assume $v_- = v_+ = 1, u_- = u_+ = 0, \theta_- = \theta_+ = 1$ in the rest of this paper.