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SPLITTING SCHEMES FOR SOME SECOND-ORDER EVOLUTIONARY EQUATIONS

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Abstract. We consider the Cauchy problem for a second-order evolutionary equation, in which the problem operator is the sum of two self-adjoint operators. The main feature of the problem is that one of the operators is represented in the form of the product of the operator A by its conjugate operator A^*. Time approximations are implemented so that the transition to a new level in time is associated with a separate solution of problems for operators A and A^*, not their products. The construction of unconditionally stable schemes is based on general results of the theory of stability (well-posedness) of operator-difference schemes in Hilbert spaces and is associated with the multiplicative perturbation of the problem operators, which lead to stable implicit schemes. As an example, the problem of the dynamics of a thin plate on an elastic foundation is considered.

Key words. Second-order evolutionary equation, Cauchy problem, explicit schemes, splitting schemes, vibrations of a thin plate.

1. Introduction

Many applied problems lead to the need for an approximate solution of the Cauchy problem for second-order evolutionary equations [2]. As a typical example, we note the dynamic problems of solid mechanics [3]. A class of problems can be distinguished, a characteristic feature of which is that the main part of the problem operator is the product of two operators. For example, when considering models of thin plates we have a biharmonic operator, the product of two Laplace operators.

Unconditionally stable schemes for these problems are built based on implicit approximations in time [5, 8]. In the theory of stability (well-posedness) of operator-difference schemes [9, 10] the most complete results were obtained on the stability of two-level and three-level schemes in Hilbert spaces. The computational complexity of solving the Cauchy problem at a new level in time using implicit schemes may be unacceptable. Therefore, various approaches are being developed to obtain computationally simpler problems when solving non-stationary problems.

Simplification of the problem at a new level is often implemented for evolutionary problems when the problem operator is represented in the form the sums of more simple operators. For such problems, additive operator-difference schemes are constructed, which are related to one or another inhomogeneous approximation in time for individual operator terms. The traditional approach is based on explicit-implicit approximations (IMEX methods) [1, 4] when one part of the problem operator is taken from the lower level in time (explicit approximation), and the other — from the upper one (implicit approximation). This idea of time-stepping is implemented most consistently when constructing splitting schemes [6, 17]. In this case, the transition to a new level in time is carried out by solving evolutionary problems for individual operator terms [19, 20].

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Another class of evolutionary problems can also be noted, in which the problem operator is represented as the product of two or more operators. An example is nonstationary problems with a variable weighting factor, the study of which is held in [10, 13]. Special time approximations are constructed to simplify the problem at a new time level. For example, paper [18] investigates schemes that are based on the solution of a discrete problem at a new time level with one operator factor.

In this paper, we consider the Cauchy problem for a second-order evolutionary equation in which the problem operator includes the product of operator A by its conjugate operator A^*. Unconditionally stable schemes are constructed based on a perturbation of both the A operator and the A^* operator. In this case, the computational implementation is associated with the separate solution of problems for operators A and A^*, not their products.

The article is organized as follows. The statement of the Cauchy problem for a second-order evolutionary equation, which includes the product of the operator A and A^*, is given in Section 2. Section 3 describes a general approach to constructing unconditionally stable schemes for second-order evolutionary equations based on multiplicative perturbation of the operators of the problem. Splitting schemes for the evolutionary problem, when the problem operator includes A^*A, are constructed in Section 4. In Section 5, our schemes are applied to the model problem of the dynamics of a thin plate on an elastic foundation. The results of our work are summarized in Section 6.

2. Problem statement

The Cauchy problem for a second-order evolutionary equation is considered in a finite-dimensional Hilbert space H. Omitting technical details, we restrict ourselves to the following homogeneous equation when

$$\frac{d^2w}{dt^2} + A^*Aw + Bw = 0, \quad 0 < t \le T, \tag{1}$$

$$w(0) = w^0, \quad \frac{dw}{dt}(0) = \widetilde{w}^0. \tag{2}$$

Assume that the operators A and B in (1) are constant (do not depend on t), and the operator B is self-adjoint and non-negative:

$$B = B^* \ge 0. \tag{3}$$

We arrive at the problem (1)-(3), for example, after discretization by spatial variables in the numerical solution of initial-boundary value problems for hyperbolic equations. The key feature of the problem under consideration is associated with the operator A, so that it enters equation (1) as the product A^*A. An example of such a construction is the biharmonic operator ($A = A^*$).

The scalar product for $u, v \in H$ is (u, v), and the norm is $\|u\| = (u, u)^{1/2}$. Let us define a Hilbert space H_S with the scalar product and norm $(u, v)_S = (Su, v)$, $\|u\|_S = (u, v)_S^{1/2}$, which is generated by the self-adjoint and positive definite operator S.

The subject of our consideration is time-stepping for equation (1). We focus on unconditionally stable schemes for an approximation solution to the problem (1)-(3), which are convenient for computational implementation. When obtaining the corresponding stability estimates we compare them with a priori estimates for the differential problem.