

Global Existence and Time-decay Rates of Solutions to 2D Magneto-micropolar Fluid Equations with Partial Viscosity

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Abstract. In this paper, we investigate the initial value problem for the two-dimensional magneto-micropolar fluid equations with partial viscosity. We prove that global existence of smooth large solutions by the energy method. Furthermore, with aid of the Fourier splitting methods, optimal time-decay rates of global smooth large solutions are also established.

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Key Words: Magneto-micropolar fluid equations with partial viscosity; global smooth large solutions; time-decay rates.

1 Introduction

In this paper, we investigate global existence and time-decay rates of smooth large solutions to two-dimensional magneto-micropolar fluid equations with partial viscosity

$$\begin{cases} \partial_t u_1 + u \cdot \nabla u_1 - B \cdot \nabla B_1 - \mu_1 \partial_y^2 u_1 - \eta \Delta u_1 + \partial_x (p + \frac{1}{2} |B|^2) - 2\eta \partial_y v = 0, \\ \partial_t u_2 + u \cdot \nabla u_2 - B \cdot \nabla B_2 - \mu_2 \partial_x^2 u_2 - \eta \Delta u_2 + \partial_y (p + \frac{1}{2} |B|^2) + 2\eta \partial_x v = 0, \\ \partial_t v - \kappa \Delta v + 4\eta v + u \cdot \nabla v - 2\eta (\partial_x u_2 - \partial_y u_1) = 0, \\ \partial_t B_1 - \nu_1 \partial_y^2 B_1 + u \cdot \nabla B_1 - B \cdot \nabla u_1 = 0, \\ \partial_t B_2 - \nu_2 \partial_x^2 B_2 + u \cdot \nabla B_2 - B \cdot \nabla u_2 = 0, \\ \partial_x u_1 + \partial_y u_2 = \partial_x B_1 + \partial_y B_2 = 0 \end{cases} \quad (1.1)$$

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with the initial value:

$$t=0: u=u_0(x,y), v=v_0(x,y), B=B_0(x,y), \quad x,y \in \mathbb{R}, \quad (1.2)$$

where $u=(u_1(x,y,t),u_2(x,y,t)), B=(B_1(x,y,t),B_2(x,y,t)) \in \mathbb{R}^2$ and $v=v(x,y,t), p=p(x,y,t) \in \mathbb{R}$ are the velocity of the fluid, magnetic field, the micro-rotational velocity and hydrostatic pressure field, respectively. μ_1 and μ_2 are kinematic viscosities, $\frac{1}{\nu_1}$ and $\frac{1}{\nu_2}$ are magnetic Reynolds numbers, η is vortex viscosity, κ is angular viscosity.

The magneto-micropolar fluid equations describes the movement of conductive micropole fluid in the magnetic field, and has been widely used in the fields of shipbuilding industry, aerospace, geophysics, hydrodynamics and meteorology, etc. The complexity of the mathematical structure of the equation has attracted extensive attention in the field of physics and mathematics. The global regularity and finite-time singularities of the large initial value solution of the three-dimensional magneto-micropolar fluid equations are still a difficult and important problem. We refer to [1–13] and [14] for existence and uniqueness of strong solutions, regularity criteria of weak solutions and blow-up criteria of smooth solutions to magneto-micropolar fluid equations.

When partial viscosities disappear, mathematical analysis of (1.1) becomes more complicated. We recall the global existence and decay estimate of solutions to the incompressible magneto-micropolar fluid equations with partial viscosity for our purpose. A few results have been established, we may refer to [15–23]. Cheng and Liu [16] studied the two-dimensional anisotropic magneto-micropolar fluid equations with vertical kinematic viscosity, horizontal magnetic diffusion and horizontal vortex viscosity and global regularity was established. Global regularity of the classical solutions for the two-dimensional magneto-micropolar equations with partial dissipation was proved by Regmi and Wu [18]. Ma [17] extended the results in [18] to other mixed partial viscosities cases and global existence and regularity were established. Shang and Gu [20] established global classical solutions to 2D magneto-micropolar equations with only velocity dissipation and partial magnetic diffusion by using the special structure of the system and the maximal regularity property of the 1D heat operator. For 3D magneto-micropolar fluid equations with mixed partial viscosity, mathematical analysis of global existence becomes more complicated. Wang and Wang [21] studied the global existence of smooth solutions for 3D magneto-micropolar fluid equations with mixed partial viscosity by energy method. Very recently, the second author of this paper and Li [22] established global well-posedness of classical small solutions and generalized the results in [23].

If $v=0$ and $\eta=0$, then the equations (1.1) is reduced to the incompressible MHD system, which describes the dynamics of electrically conducting fluids arising from plasmas, liquid metals, salt water or some other physical phenomena. It is well known that MHD system is the incompressible Navier-Stokes equations coupled with the magnetic fields. Hence, it is not surprising that the question of finite time singularity or global regularity is an outstanding open problem in the mathematical fluid mechanics. Due to the importance and challenge of the problem on global regularity and finite-time singularities of