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NONEXISTENCE OF POSITIVE SOLUTIONS FOR A FOUR-POINT BOUNDARY VALUE PROBLEM FOR FRACTIONAL DIFFERENTIAL EQUATION*†

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Abstract

In this paper, we investigate the nonexistence of positive solutions for a class of four-point boundary value problem of nonlinear differential equation with fractional order derivative. We give sufficient conditions on nonlinear term and the parameter such that the boundary value problem has no positive solutions. Some examples are presented to illustrate the main results.

Keywords positive solution; fractional differential equation; fixed point; cone

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1 Introduction

In this paper, we consider the nonexistence of the positive solution for the following boundary value problem of differential equation involving the Caputo's fractional order derivative

$$D^{\alpha}_{0+}u(t) + \lambda f(t, u(t)) = 0, \quad t \in (0, 1), \tag{1.1}$$

$$u'(0) - \mu_1 u(\xi) = 0, \quad u'(1) + \mu_2 u(\eta) = 0, \tag{1.2}$$

where $1 < \alpha \leq 2,\ 0 \leq \xi \leq \eta \leq 1,\ 0 \leq \mu_1, \mu_2 \leq 1$ and satisfy the following conditions:

(H1) $\Delta = \mu_1(1 + \mu_2\eta - \mu_2\xi) + \mu_2 < (\alpha - 1)(1 - \mu_1\xi)$;

(H2) $f \in C([0,\ 1] \times R^+, R^+)$.

Due to the development of the theory of fractional calculus and its applications, such as in the fields of physics, electro-dynamics of complex medium, control theory,

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Bode's analysis of feedback amplifiers, blood flow phenomena, aerodynamics and polymer rheology, electron-analytical chemistry, etc, many works on fractional calculus, fractional order differential equations have appeared [1-7]. Recently, there have been many results concerning the solutions and positive solutions for boundary value problems for nonlinear fractional differential equations, see [8-29] and references therein.

For example, Bai and Lü [12] considered the following Dirichlet boundary value problem of fractional differential equation

$$D_{0+}^{\alpha}u(t)+f(t,u(t))=0,\quad t\in(0,1), \tag{1.3}$$

$$u(0)=0=u(1),\quad 1<\alpha\le 2. \tag{1.4}$$

By means of different fixed-point theorems on a cone, some existence and multiplicity results of positive solutions were obtained. Jiang and Yuan [20] improved the results in [12] by discussing some new positive properties of the Green function for problem (1.3). By using the fixed point theorem on a cone due to Krasnoselskii, the authors established the existence results of positive solution for problem (1.3). Recently, Caballero et al. [21] obtained the existence and uniqueness of positive solution for singular boundary value problem (1.3). The existence results were established in the case that the nonlinear term f may be singular at $t=0$.

There are also some results concerning multi-point boundary value problems for differential equations of fractional order.

Wang et al. [25] considered the boundary value problem of fractional differential equation with integral condition

$$D_{0+}^{\alpha}u(t)+q(t)f(t,u(t))=0,\quad t\in(0,1),\ n-1<\alpha<n, \tag{1.5}$$

$$u(0)=u'(0)=\cdots=u^{(n-2)}(0)=0,\quad u(1)=\int_0^1 u(s)\mathrm{d}A(s), \tag{1.6}$$

where $\alpha>2$, $\int_0^1 u(s)\mathrm{d}A(s)$ was given by Riemann-Stieltjes integral with a signed measure. By using the fixed point theorem, the existence of positive solution for this problem were established.

Many works deal with the existence and multiplicity of positive solution for fractional differential equation (1.1) under the boundary conditions (1.2). Zhao, Chai and Ge [28] considered a class of four-point fractional boundary value problem of the form

$$D_{0+}^{\alpha}u(t)+f(t,u(t))=0,\quad t\in(0,1), \tag{1.7}$$

$$u'(0)-\mu_1u(\xi)=0,\quad u'(1)+\mu_2u(\eta)=0, \tag{1.8}$$

where $1<\alpha\le 2,\ 0\le\xi\le\eta\le 1,\ 0\le\mu_1,\mu_2\le 1$ with the condition