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Study of Stability Criteria of Numerical Solution of Ordinary and Partial Differential Equations Using Eulers and Finite Difference Scheme

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Abstract. In this paper we have discussed solution and stability analysis of ordinary and partial differential equation with boundary value problem. We investigated periodic stability in Eulers scheme and also discussed PDEs by finite difference scheme. Numerical example has been discussed finding nature of stability. All given result more accurate other than existing methods.

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1 Introduction

Mathematics is a language which can describe patterns in everyday life as well as abstract concepts existing only in our minds. Patterns exist in data, functions, and sets constructed around a common theme, but the most tangible patterns are visual. Applications of mathematical concepts are also rich sources of visual aids to gain perspective and understanding. Differential equations are a natural way to model relationships between different measurable phenomena. Differential equations do just thatafter being developed to adequately match the dynamics involved. For instance, we are interested in how fast different parts of a frying pan heat up on the stove. Derived from simplifying assumptions about density, specific heat, and the conservation of energy, the heat equation will do just that! I we use the heat equation called a test equation, as a control in our investigations of more complicated partial differential equations (PDEs). [1–3]

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2 Stability analysis of single step method

The analytical solution $u(t_j)$ of the differential equation, the difference solution u_j of the difference equation and the numerical solution $\bar{u}_j$ can be related by a relation of the form

$$|u(t_j)-\bar{u}|\leq|u(t_j)-u_j|+|u_j-\bar{u}|.$$

We find that this difference depends on the values $|u(t_j)-u_j|$ and $|u_j-\bar{u}|$. The value $|u(t_j)-u_j|$ is the truncation error which arises because the differential equation is replaced by the difference equation. A method is said to be consistent if it is at least of order 1. For a consistent method, the truncation error tends to zero as h approaches zero. The numerical error $|u_j-\bar{u}|$ arises because in actual computation, we cannot compute the difference solution exactly as we are faced wit the round off errors. In fact, in some cases the numerical solution may differ considerable from the difference solution. If the effect of the total error including the round-off error remain bounded as $j\to\infty$ with fixed step size, then the difference method is said to be stable, otherwise unstable [4–6]. Consider the equation

$$u=\lambda u,\quad u(t_0)=u_0,$$

where λ may be real or complex number. The analytical solution of the equation satisfies the equation

$$u(t_{j+1})=e^{\lambda h}u(t_j).$$

If we apply any single step method to solve the test equation then we get a first order difference equation of the form

$$u_{j+1}=E(\lambda h)u_j,\quad j=0,1,2,\dots .$$

Let $\epsilon_j=u_j-u(t_j)$, then we obtain

$$\epsilon_{j+1}=[E(\lambda h)-e^{\lambda h}]u(t_j)+E(\lambda h)\epsilon_j.$$

The first term of the right hand side is the local truncation error and the second term on the right hand side is the error propagated from the step t_j to t_{j+1}. The error on the next step t_{j+2} satisfies the equation,

$$\epsilon_{j+2}=[E^2(\lambda h)-e^{2\lambda h}]u(t_j)+E^2(\lambda h)\epsilon_j,$$

where again first term of the right hand side is the local truncation error and the second term on the right hand side is the propagated error. The errors in computations do not grow, if the propagated error tends to zero or is at least bounded.We call a single step method, when applied to the test equation $u=\lambda u$.

Absolutely stable if $|E(\lambda h)|\leq 1,\quad \lambda<0.$

Relatively stable if $|E(\lambda h)|\leq e^{\lambda h},\quad \lambda>0.$