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TOTAL VARIATION BASED PURE QUATERNION DICTIONARY LEARNING METHOD FOR COLOR IMAGE DENOISING

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Abstract. As an important pre-processing step for many related computer vision tasks, color image denoising has attracted considerable attention in image processing. However, traditional methods often regard the red, green, and blue channels of color images independently without considering the correlations among the three channels. In order to overcome this deficiency, this paper proposes a novel dictionary method for color image denoising based on pure quaternion representation, which efficiently deals with both single-channel and cross-channel information. The pure quaternion constraint is firstly used to force the sparse representations of color images to contain only red, green, and blue color information. Moreover, a total variation regularization is proposed in the quaternion domain and embedded into the pure quaternion-based representation model, which is effective to recover the sharp edges of color images. To solve the proposed model, a new numerical scheme is also developed based on the alternating minimization method (AMM). Experimental results demonstrate that the proposed model has better denoising results than the state-of-the-art methods, including a deep learning approach DnCNN, in terms of PSNR, SSIM, and visual quality.

Key words. Color image denoising, singular value decomposition, pure quaternion matrix, total variation, sparse representation.

1. Introduction

Color image denoising is a fundamental image processing task that focuses on obtaining a clean color image from a noisy observation [39]. Color images have been widely used in many fields, from medical imaging to automatic driving [15, 47, 53] Generally speaking, a color image contains red, blue, and green (RGB) channels, which are highly related to the image [18]. As a matter of fact, each pixel x of color image contains three gray pixels, i.e., $x = (x_r, x_g, x_b)$, where x_r, x_g, and x_b are RGB channels respectively. With a little changes of any channel, the color of x will have corresponding effects. The phenomenon of image degradation resulting from noise adversely affects the subsequent image processing and analysis, and visual effects [23, 26, 25]. Therefore, noise suppressing for improving color image quality is an essential process for many imaging tasks [37]. In this paper, we focus on the problem of removing additive Gaussian noise in color images. Mathematically, the degraded image $\mathbf{Y} \in \mathbb{R}^{m\times n}$ can be formulated as

$$\mathbf{Y} = \mathbf{X} + \mathbf{W}, \tag{1}$$

where $\mathbf{X} \in \mathbb{R}^{m\times n}$ is the original image, and $\mathbf{W} \in \mathbb{R}^{m\times n}$ is the Gaussian white noise. In the past decades, many excellent denoising methods have been proposed, such as dictionary learning method [19], nonlocal means [3], block-matching and 3D filtering [9], and total variation [45, 46, 51], etc. We refer the reader to see [16] for a comprehensive review of the image denoising.

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Among the various denoising techniques, the dictionary-based method generalized K-means clustering for singular value decomposition (K-SVD) shows its superiority in reserving the textures, therefore, it has attracted considerable improvements in the last decade [11]. Indeed, Elad and Aharon [1] firstly proposed the effective patch-based method with K-SVD algorithm via sparse representation over a learned dictionary and updated the coefficients with orthogonal matching pursuit (OMP) algorithm. Given the noisy observation $\mathbf{Y}$, their model can be expressed as

$$\min_{\mathbf{D},\mathbf{a}_{ij},\mathbf{X}} \lambda\|\mathbf{X}-\mathbf{Y}\|_2^2 + \sum_{i,j}(\mu_{ij}\|\mathbf{a}_{ij}\|_0 + \|\mathbf{D}\mathbf{a}_{ij} - \mathcal{R}_{ij}\mathbf{X}\|_2^2), \tag{2}$$

where $\mathbf{D} \in \mathbb{R}^{m\times k}$ is the dictionary matrix, the $[i,j]$ indicates the image patch location, $\mathcal{R}_{ij}$ is an operator extracting the square $\sqrt{n}\times\sqrt{n}$ patch from the image at position $[i,j]$, and the vector $\mathbf{a}_{ij} \in \mathbb{R}^{k\times 1}$ is the coefficient vector for the corresponding patch with $\|\cdot\|_0$ being the ℓ_0-norm to count the nonzero number in the vector. As this method is designed for gray images initially, it will generate color distortion while be applied to color images by dealing with the three channels independently [50]. Hence, the patch-based dictionary method was improved to the patch group-based dictionary methods [48], which can eliminate the color bias. However, they still ignore the relationship among the color channels [50].

Recently, the quaternion representation has obtained much attention in image processing. The quaternion represents a color pixel by a structure, which can integrate the information of three channels. This advantage has promoted the application of quaternion representation in the color image processing [24]. For example, Yu et al. [54] applied quaternion-based weighted nuclear norm minimization (QWNNM) for color image denoising. The QWNNM model achieves better results than the real value-based weighted nuclear norm minimization method. Wang et al. [42] handled the color image segmentation with the quaternion-based method and has better results than the real value-based methods. Denoting a dot in variances as quaternion number and $\mathbb{H}$ as quaternion domain, the quaternion-based degradation model for color noise is given as

$$\dot{\mathbf{Y}} = \dot{\mathbf{X}} + \dot{\mathbf{W}}, \tag{3}$$

where $\dot{\mathbf{Y}}$, $\dot{\mathbf{X}}$, and $\dot{\mathbf{W}} \in \mathbb{H}^{m\times n}$ are the noisy image, latent clear image, and Gaussian white noise with zero mean and standard variance σ of quaternion form, respectively. The detailed information about quaternion please see Section 2.2. Comparing with vector-based models, the quaternion-based models fully utilize the relationship between channels and the orthogonal property for the coefficients of different channels [6] and thus generate better results. Due to the superiority of the quaternion-based method, Xu et al. [50] improved the model (2) with quaternion representation, and called it the K-QSVD model. Their idea is to fit color images with quaternion matrices and train the dictionary with the K-QSVD[1] and the QOMP[2] algorithms. Their K-QSVD model is formulated as follows

$$\min_{\dot{\mathbf{D}},\dot{\mathbf{a}}_{ij},\dot{\mathbf{X}}} \lambda\|\dot{\mathbf{X}}-\dot{\mathbf{Y}}\|_2^2 + \sum_{i,j}(\mu_{ij}\|\dot{\mathbf{a}}_{ij}\|_0 + \|\dot{\mathbf{D}}\dot{\mathbf{a}}_{ij} - \dot{\mathcal{R}}_{ij}\dot{\mathbf{X}}\|_2^2), \tag{4}$$

where $\dot{\mathbf{D}} \in \mathbb{H}^{m\times k}$ is the dictionary matrix in quaternion form, the indicator $[i,j]$ marks the patch location, $\dot{\mathcal{R}}_{ij}$ is an operator extracting the square $\sqrt{n}\times\sqrt{n}$ patch

[1]The K-QSVD algorithm is the extension of the K-SVD algorithm, with all algebra operations in quaternion system.

[2]The QOMP algorithm is the extension of the OMP algorithm, with all algebra operations in quaternion system.