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VALIDATION AND VERIFICATION OF TURBULENCE MIXING DUE TO RICHTMYER-MESHKOV INSTABILITY OF AN AIR/SF$_6$ INTERFACE

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Abstract. Turbulent mixing due to hydrodynamic instabilities occurs in a wide range of science and engineering applications such as supernova explosions and inertial confinement fusion. The experimental, theoretical and numerical studies help us to understand the dynamics of hydrodynamically unstable interfaces between fluids in these important problems. In this paper, we present an increasingly accurate and robust front tracking method for the numerical simulations of Richtmyer-Meshkov Instability (RMI) to estimate the growth rate. The single-mode shock tube experiments of Collins and Jacobs 2002 [1] for two incident shock strengths ($M = 1.11$ and $M = 1.21$) are used to validate the RMI simulations. The simulations based on the classical fifth order weighted essentially non-oscillatory (WENO) scheme of Jiang and Shu [2] with Yang's artificial compression [3] are compared with Collins and Jacobs 2002 shock tube experiments. We investigate the resolution effects using front tracking with WENO schemes on the two-dimensional RMI of an air/SF$_6$ interface. We achieve very good agreement on the early time interface displacement and amplitude growth rate between simulations and experiments for Mach number $M = 1.11$. A 4% discrepancy on early-time amplitude is observed between the fine grid simulation and the $M = 1.21$ experiments of Collins and Jacobs 2002.

Key words. Turbulent mixing, Rayleigh-Taylor instability, Richtmyer-Meshkov instability, front tracking, weighted essentially non-oscillatory scheme.

1. Introduction

Turbulent mixing due to hydrodynamic instabilities occurs in many scientific and engineering applications. The formation of gravitational induced mixing in oceanography; supernovae explosions in astrophysics, and the performance assessment for inertial confinement fusion (ICF) are ideal sub-problems to study and understand the dynamics of turbulence and mixing [4]. In the dynamics of turbulence, hydrodynamic instabilities of fluid flows such as Kelvin–Helmholtz, Rayleigh–Taylor, Richtmyer–Meshkov are observed. The review papers of Zhou [5, 6], and Abarzhi, Gauthier and Sreenivasan [7] provide detailed resource information on the theory, experiment and computations of these important physical instabilities. While the velocity difference at the interface between two fluids develops the Kelvin–Helmholtz instability (KHI), the density difference with constant and impulsive acceleration develops the Rayleigh–Taylor instabilitiy (RTI) and Richtmyer–Meshkov instabilitiy (RMI) respectively. RTI arises at the perturbed interface between two fluids of different densities whenever the pressure gradient opposes the density gradient. RMI arises when a shock wave interacts with the perturbed interface. RMI is also known as impulsive or shock-induced RTI. An overview of RTI and RMI and the effects of material strengths, chemical reactions and magnetic fields, as well as the role of the instabilities in scientific and engineering applications can be found in two review articles [8, 9]. The evolution of the perturbed interface development and the interaction between the fluids at the macro/meso/micro length scales have been the main interest of researchers.

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The motions of fluid flows are described by the Euler equations, a system of partial differential equations, where the effect of molecular processes are neglected. The more general equations are Navier-Stokes equations (NSE). These problems are deeply multiscale and the level of scales that are desired to be resolved identify the characteristic properties. The three numerical approaches to model turbulence are (i) Direct Numerical Simulation (DNS) [10], the full NSE are resolved without any models for turbulence, (ii) Large Eddy Simulation (LES), the flow field is resolved down to a certain length scale and scales smaller than that are modeled rather than resolved, and (iii) Reynolds-Averaged Navier-Stokes (RANS), the time-averaged equations are solved for mean values of all quantities. In the LES family, the implicit LES (ILES) solves the governing equations using an implicit subgrid scale model. ILES assumes that small and unresolved scales are purely dissipative and the numerical discretization errors are the source of artificial dissipation [11]. These approaches are briefly reviewed in section 2.1 and a recent review article summarizes in detail [12]. Within these approaches, the most accurate DNS has the highest computational cost and the least accurate RANS is the most desired approach in complex engineering applications due to the low computational cost.

There are hydrodynamic codes such as *CFDNS, HYDRA, Miranda, RAGE, RAPTOR, TURMOIL,* and FronTier that have been under continuous development to study the RTI/RMI induced flow, turbulence and mixing to predict the growth rate of the mixing zone accurately based on the mathematical and numerical frameworks [5, 6]. *CFDNS* is a Los Alamos National Laboratory (LANL) hydrodynamic simulation code designed for direct numerical simulation of turbulent flows [13]. *HYDRA* is based on arbitrary Lagrangian-Eulerian (ALE) mesh used for the numerical simulation of instabilities in ICF laser-driven hohlraum [14]. *Miranda* is a Lawrence Livermore National Laboratory (LLNL) hydrodynamic simulation code designed for large-eddy simulation of multicomponent flows with turbulent mixing [15]. The spectral, high-order compact scheme with local artificial viscosity, diffusivity is used in order to remove oscillations and capture shocks and contact discontinuities [16]. *RAGE* is a 'radiation adaptive grid Eulerian' radiation-hydrodynamic code in which the hydrodynamics is a basic Godunov solver [17]. *RAPTOR* is a hydrodynamic code based on a Godunov-type finite volume method that solves Riemann problem at cell interface using an adaptive mesh refinement technique [18]. *TURMOIL* is a Lagrange-remap hydrocode which calculates the mixing of compressible fluids by solving the Euler equations plus advection equations for fluid mass fractions [19]. FronTier is based on front tracking method for accurate representation of the interface [20]. In this paper, we present the algorithmic features of FronTier that are used to study the two-dimensional RMI instabilities. The front tracking algorithm is a way to track the interface explicitly with high order accuracy. It is a unique method demonstrated to avoid systematic errors in an important class of problems revolving around turbulent mixing [21, 22, 23]. This technique stores and dynamically evolves a meshed front that partitions a simulation domain into two or more regions, each representing a different material or physics model (see section 2.2).

The incident shock strength characterized by Mach number has a big effect on the dynamics of flows. Some numerical methods such as filtered spectral methods only show numerically stable solutions under the moderate Mach number. When the Mach number is large, the methods become non-robust. Robust numerical methods based on solution-averaged or solution-reconstruction methods also known as reconstruction-evolutionary methods are used in the hydrodynamically unstable flow