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Physics-Driven Learning of the Steady Navier-Stokes Equations using Deep Convolutional Neural Networks

Hao Ma[1], Yuxuan Zhang[2], Nils Thuerey[3], Xiangyu Hu[4,*] and Oskar J. Haidn[1]

[1] *Department of Aerospace and Geodesy, Technical University of Munich, 85748 Garching, Germany.*
[2] *Beijing Aerospace Propulsion Institute, 100076, Beijing, China.*
[3] *Technical University of Munich, 85748, Garching, Germany.*
[4] *Department of Mechanical Engineering, Technical University of Munich, 85748 Garching, Germany.*

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Abstract. Recently, physics-driven deep learning methods have shown particular promise for the prediction of physical fields, especially to reduce the dependency on large amounts of pre-computed training data. In this work, we target the physics-driven learning of complex flow fields with high resolutions. We propose the use of *Convolutional neural networks* (CNN) based U-net architectures to efficiently represent and reconstruct the input and output fields, respectively. By introducing Navier-Stokes equations and boundary conditions into loss functions, the physics-driven CNN is designed to predict corresponding steady flow fields directly. In particular, this prevents many of the difficulties associated with approaches employing fully connected neural networks. Several numerical experiments are conducted to investigate the behavior of the CNN approach, and the results indicate that a first-order accuracy has been achieved. Specifically for the case of a flow around a cylinder, different flow regimes can be learned and the adhered "twin-vortices" are predicted correctly. The numerical results also show that the training for multiple cases is accelerated significantly, especially for the difficult cases at low Reynolds numbers, and when limited reference solutions are used as supplementary learning targets.

AMS subject classifications: 76D05

Key words: Deep learning, physics-driven method, convolutional neural networks, Navier-Stokes equations.

*Corresponding author. *Email addresses:* hao.ma@tum.de (H. Ma), uruz7@live.com (Y. Zhang), nils.thuerey@tum.de (N. Thuerey), xiangyu.hu@tum.de (X. Hu), oskar.haidn@tum.de (O. J. Haidn)

1 Introduction

In some practical fluid mechanics problems such as real-time or frequent query analysis, a large number of solutions for different initial/boundary condition combinations are to be considered [1–3]. For the traditional discrete analysis, numerical simulations have to be conducted repeatedly and the computational cost quickly becomes overly expensive [4]. In contrast to classical computational methods, machine learning approaches, and especially the field of deep learning that employs *Neural Networks* (NN), have demonstrated their capabilities to predict flow fields rapidly and accurately [5–7].

The previous research on flow field prediction using NN is mainly focused on data-driven methods. Besides the indirect way using closure model [8, 9], the field solution can also be directly obtained from the network trained with a large number of samples [10, 11]. However, for complex flows in practical engineering problems, the training samples very often require extraction, pre-processing and may be hard to obtain [12]. Some data-driven learning work utilizes *Computational Fluid Dynamics* (CFD) approach to generate the data sets [13–15], and it does not really solve the demand of avoiding the big computational cost of discrete methods.

In order to remedy the above-mentioned shortcomings, physics-driven methods are a relatively new development. By providing physics information, NN are able to directly obtain the field solution with much less or even no training data. Based on *Multi-layer Perceptron* (MLP) [16], Raissi et al. designed a *Physics Informed Neural Networks* (PINN). By the constraint of loss function employing *Partial Differential Equations* (PDEs), the outputs gradually approach the ones obeying the physics laws [17–19]. However, due to the full connectivity between the neurons, MLP suffer from extensive memory requirements and statistical inefficiencies [20]. Therefore, it is difficult to handle well the multi-dimensional learning space with high-resolution physics fields containing much more details. Taking the highest resolution solution in Ref. [21] as an example, the MLP with one single temperature channel has over 1 million weights. In addition, there are attempts [22] to build surrogates to predict the physics solutions for more complex fluid dynamics problems with parameterized fluid properties and objective geometries. Again, these parametric variables constitute a high dimensional input space and prevents to establish an accurate map for evaluating the corresponding flow fields. One avenue for alleviating this problem is to employ the reduced-order modeling to compress and reconstruct the flow fields apart from training the network [10,23]. However, this operation not only is complicated but also may introduce additional errors from the projection onto reduced space [24]. In addition, MLP architecture by itself does not take into account the spatial structure of data. The data points in the learning domain irrespective of their distance are treated in a same way [25]. However, the physics laws represented with PDEs are based on the localities of data points, which suggests that the capability of NN to reflect this spatial relationship can be very important, especially for the physics-driven methods which are constrained only by PDEs.

On the other hand, *Convolutional Neural Networks* (CNN) represent a specialized and