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Global Well-Posedness for the 3D Tropical Climate Model without Thermal Diffusion

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Abstract. In this paper, we consider the Cauchy problem of 3D tropical climate model with zero thermal diffusion. Firstly, we establish the global regularity for this system with fractional diffusion $\alpha=\beta=5/4$. Secondly, by adding only a damp term, we obtain the global well-posedness for small initial data.

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1 Introduction

This paper focuses on the following tropical climate model (TCM) given by

$$\begin{cases} \partial_t u+u\cdot\nabla u+\mu\Lambda^{2\alpha}u+\nabla\Pi+\mathrm{div}(v\otimes v)=0, & x\in\mathbb{R}^d,\ t>0,\\ \partial_t v+u\cdot\nabla v+\nu\Lambda^{2\beta}v+v\cdot\nabla u+\nabla\theta=0, & x\in\mathbb{R}^d,\ t>0,\\ \partial_t\theta+u\cdot\nabla\theta+\eta\Lambda^{2\gamma}\theta+\mathrm{div}v=0, & x\in\mathbb{R}^d,\ t>0,\\ \mathrm{div}u=0, & x\in\mathbb{R}^d,\ t\geq 0,\\ (u,v,\theta)|_{t=0}=(u_0,v_0,\theta_0), & x\in\mathbb{R}^d, \end{cases} \tag{1.1}$$

here $\mu,\nu,\eta,\alpha,\beta,\gamma$ are non-negative parameters, $u=u(t,x)$ and $v=v(t,x)$ stand for the barotropic mode and the first baroclinic mode of the vector velocity, respectively. $\Pi=$

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$\Pi(t,x)$ and $\theta=\theta(t,x)$ denote the scalar pressure and scalar temperature, respectively. $\Lambda=\sqrt{-\Delta}$ is the Zygmund operator and the fractional power operator $\Lambda^{\delta}=(-\Delta)^{\frac{\delta}{2}}$ is defined through Fourier transform as

$$\mathcal{F}(\Lambda^{\delta}f)(\xi)=|\xi|^{\delta}\mathcal{F}f(\xi).$$

We make the convention that by $\delta=0$ we mean that there is no dissipation.

The original version of (1.1) without any fractional Laplacian terms was derived by Frierson–Majda–Pauluis [1] from the inviscid primitive equations with the aid of performing a Galerkin truncation to the hydrostatic Boussinesq equations, of which the first baroclinic mode had been originally used in some studies of tropical atmosphere. More relevant background on the tropical climate model can be found in [2–4] and the references therein.

From the mathematical point of view, the tropical climate model (1.1) are significant generalizations of the generalized magnetohydrodynamic (GMHD) equations which model the complex interaction between the fluid dynamic phenomena. In fact, when the temperature $\theta\equiv$ Constant, (1.1) is reduced to the GMHD equations (see e.g., [5])

$$\begin{cases}\partial_t u+u\cdot\nabla u+\nabla\Pi+\mu\Lambda^{2\alpha}u-v\cdot\nabla v=0, & x\in\mathbb{R}^d,\ t>0,\\ \partial_t v+u\cdot\nabla v+\nu\Lambda^{2\beta}v-v\cdot\nabla u=0, & x\in\mathbb{R}^d,\ t>0,\\ \mathrm{div}u=\mathrm{div}v=0, & x\in\mathbb{R}^d,\ t\geq 0,\\ (u(0,x),v(0,x))=(u_0(x),v_0(x)), & x\in\mathbb{R}^d,\end{cases}\tag{1.2}$$

here $u=u(t,x)$ and $v=v(t,x)$ denote the velocity and the magnetic field of the fluid, respectively.

The mathematical studies on TCM have attracted considerable attention recently from various authors and have motivated a large number of research papers (see [6–11]). Next, we mainly recall some global well-posedness results which are more relative with our research in this field. Li–Titi [12] introduced a new quantity to bypass the obstacle caused by the absence of thermal diffusion and proved the global well-posedness of strong solutions for the 2D TCM with $\alpha=\beta=1$ and $\mu>0,\nu>0,\eta=0$. Later, Ye [13] obtained the global regularity of a tropical climate model with the very weak dissipation of the barotropic ($\alpha>0,\beta=\gamma=1$ and $\mu,\nu,\eta>0$) by the "weakly nonlinear" energy estimates approach and the maximal $L^q_tL^p_x$ regularity for the heat kernel. Dong et al. [14] established the global regularity results for the 2D system (1.1) without thermal diffusion with $\alpha+\beta=2,\beta\in(1,\frac{3}{2}],\mu,\nu>0$ and $\alpha=2,\mu>0,\nu=\eta=0$, respectively. Recently, Dong et al. [15] established the global existence and regularity for the 2D system (1.1) ($\mu,\nu,\eta>0,\beta=1$) with the fractional dissipation which are in two very broad ranges, namely,

$$\gamma\geq\frac{4+\alpha-\sqrt{\alpha^2+8\alpha+8}}{2},\qquad \text{if}\quad 0<\alpha<\frac{1}{2};\quad \gamma\geq 1-\alpha,\quad \text{if}\quad \frac{1}{2}\leq\alpha\leq 1.$$