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Optimal Decay Rates of Solutions to a Blood Flow Model

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Abstract. In this paper, we are concerned with the asymptotic behavior of solutions to Cauchy problem of a blood flow model. Under some smallness conditions on the initial perturbations, we prove that Cauchy problem of blood flow model admits a unique global smooth solution, and such solution converges time-asymptotically to corresponding equilibrium states. Furthermore, the optimal convergence rates are also obtained. The approach adopted in this paper is Green's function method together with time-weighted energy estimates.

AMS subject classifications: 85A25, 35L65, 35B40

Key words: Asymptotic behavior, blood flow model, Green's function method, time-weighted energy estimates.

1 Introduction

Cardiovascular disease is a common disease that seriously threatens the health of human beings. It is characterized by high prevalence rate, high disability rate and high mortality rate. Every year, there are approximately 15 million people die of cardiovascular disease, ranking first among all causes of death. So it is urgent for researchers to develop models and methods for prevention and treatment. To understand the fundamental mechanisms of this complex physiological system, numerous mathematical models were initiated in the 1950's. Among them, the hyperbolic PDE [10, 18] has attracted considerable attentions in recent years

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$$\begin{cases} A_t + m_x = 0, \\ m_t + \left(\alpha \dfrac{m^2}{A}\right)_x + \dfrac{A}{\rho} p(A)_x = -\mu \dfrac{m}{A}, \end{cases} \tag{1.1}$$

where $(x,t) \in \mathbb{R} \times \mathbb{R}^+$. Here, $A = A(x,t) \geq 0$ denotes cross-sectional area, $m = m(x,t)$ is the flow rate. $\rho > 0$ denotes fluid density, $\alpha \geq 1$ is the ratio of the averaged axial momentum and $\mu > 0$ is proportional to the viscosity of the fluid. It should be noted that the last three terms are assumed to be constant throughout this paper. Moreover, the pressure $p(A)$ is expressed by

$$p(A) = P_{ext} + G_0 \left(\left(\frac{A}{A_r} \right)^{\frac{\beta}{2}} - 1 \right) + \frac{\nu}{A_r} (\sqrt{A})_t, \tag{1.2}$$

where the constants $P_{ext} \geq 0$ denotes the constant external pressure, $A_r > 0$ is the reference cross-sectional area, $G_0 > 0$ describes the stiffness of the vessel wall, $\nu \geq 0$ denotes viscoelastic coefficient and $\beta > 0$ captures the linearity/nonlinearity of the stress-strain response.

As an important biological model, (1.1) was used to describe the complicated physiological phenomena with human vascular system, since its initiation, it has attracted considerable attentions and have been studied in a wide range of aspects. Here we only focus on the related work on the existence and stability of solutions. When $\nu = 0$, (1.1) reduces to a quasilinear hyperbolic system. Čanić and Kim [3] studied the existence of global-in-time regular solutions under some smallness conditions. Later Li and Čanić [11] illuminated the influence of the viscous damping μ on the solutions to the Cauchy problem of this model. They pointed out that for $\mu > 0$, the data classes that produce smooth solutions are richer than $\mu = 0$, and for the physiologically relevant data that give rise to shock formation, when $\mu > 0$, shock formation is delayed in time. Furthermore, Li and Zhao [12] showed the initial-boundary value problem of the blood flow model admits a unique global smooth solution for small initial data, and such solution converges to constant equilibrium states exponentially as time goes to infinity due to viscous damping and boundary effects. Subsequently, the authors studied the same type of asymptotic states of L^∞ entropy weak solution for large, rough initial data containing vacuum in [13]. In addition, the coupling of a quasilinear hyperbolic system with a Windkessel type boundary was considered in [4]. When considering $\nu > 0$, the viscous term makes the system of hyperbolic/parabolic nature. Even so, the hyperbolic nature of the system (1.1) is still dominant in the blood flow model. Recently, Maity [14] showed that the existence and uniqueness of maximal strong solutions of the system. For other results concerning numerical simulations of this model, we refer to the interesting works [1, 2, 5, 17, 22, 23].