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A SSLE-TYPE ALGORITHM OF QUASI-STRONGLY SUB-FEASIBLE DIRECTIONS FOR INEQUALITY CONSTRAINED MINIMAX PROBLEMS *

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Abstract

In this paper, we discuss the nonlinear minimax problems with inequality constraints. Based on the stationary conditions of the discussed problems, we propose a sequential systems of linear equations (SSLE)-type algorithm of quasi-strongly sub-feasible directions with an arbitrary initial iteration point. By means of the new working set, we develop a new technique for constructing the sub-matrix in the lower right corner of the coefficient matrix of the system of linear equations (SLE). At each iteration, two systems of linear equations (SLEs) with the same uniformly nonsingular coefficient matrix are solved. Under mild conditions, the proposed algorithm possesses global and strong convergence. Finally, some preliminary numerical experiments are reported.

Mathematics subject classification: 90C30, 90C47, 65K05.
Key words: Inequality constraints, Minimax problems, Method of quasi-strongly sub-feasible directions, SSLE-type algorithm, Global and strong convergence.

1. Introduction

The minimax optimization may occur in engineering design [1], control system design [2], portfolio optimization [3], or as subproblems of algorithms for in semi-infinite minimax problems [4]. In this work, we discuss the nonlinear minimax problem with inequality constraints of the form

$$\begin{array}{ll} \min & F(x) \\ \text{s.t.} & f_i(x) \le 0, \quad i \in J = \{m+1, \cdots, m+l\}, \end{array} \tag{1.1}$$

where $F(x) = \max\{f_i(x), i \in I\}$ with $I = \{1, 2, \cdots, m\}$, and $f_i(x)(i \in I \cup J): \mathbb{R}^n \to \mathbb{R}$. Since the objective function of this minimax problem is continuous but non-differentiable, the classical methods of smooth nonlinear programming cannot be used directly to solve the problem. Fortunately, by introducing an additional variable, a minimax problem can be equivalently reformulated as a smooth nonlinear programming, and many algorithms have been proposed,

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such as the sequential quadratic programming type methods [5–7], the interior point algorithm [8], the exponential smoothing algorithm [9], the trust-region method [10] and the sequential quadratically constrained quadratic programming algorithm [11].

It is known that the SSLE method (also call QP-free algorithm) is one of the effective methods for solving smooth nonlinear constrained optimization. In 1988, based on the KKT conditions, Panier, Tits and Herskovits [12] proposed a feasible QP-free algorithm for inequality constrained optimization, where two linear systems with a same coefficient matrix and a least squares subproblem need to be solved at each iteration. To overcome the calculation of the least squares subproblem, Yang, Li and Qi [13] proposed a feasible SSLE algorithm, where three reduced linear systems need to be solved at each iteration. Furthermore, to improve the convergence properties and numerical performance, many efforts have been made for research on SSLE-type (or QP-free) algorithms, in Refs. [14–18]. In fact, a feasible point is required to initialize the algorithm for the methods of feasible direction [6, 11–15], to overcome such kind of difficulty in a more general context, Jian and his collaborators proposed a method of strongly sub-feasible directions (MSSFD), see [18–20] and [21, Chapter 2]. The main features of the MSSFD can be described as follows: the initial point can be chosen arbitrarily without using any penalty parameters or penalty functions; the operations of initialization (Phase I) and optimization (Phase II) can be well unified automatically; the feasibility of a constraint is maintained through the iterations once it is reached, and therefore the number of feasible constraints is nondecreasing.

Recently, by improving the MSSFD, Jian et al. [22] presented the method of quasi-strongly sub-feasible directions (MQSSFD), the main characteristic is that the request $I^-(x^k) \subseteq I^-(x^{k+1})$ in MSSFD is relaxed by $|I^-(x^k)| \leq |I^-(x^{k+1})|$, where $|I^-(x^k)|$ means the number of functions satisfying the inequality constraints at x^k, thus the step size yielded by MQSSFD is larger than MSSFD. Furthermore, combining the idea of MQSSFD, Ma and Jian [16] presented a new QP-free algorithm for inequality constrained optimization with an arbitrary initial iteration point. At each iteration, this algorithm solves only two SLEs with a same uniformly nonsingular coefficient matrix to obtain the search direction. And the QP-free algorithm possesses nice theoretical properties and numerical results. In addition, the superiority of numerical performance for MQSSFD has also been verified by norm-relaxed sequential quadratic programming method in [23].

In this work, motivated by the idea of MQSSFD and QP-free algorithm for nonlinear inequality constrained optimization in Ref. [16], and based on the stationary point conditions of the nonlinear minimax problems with inequality constraints, we propose a SSLE-type algorithm of quasi-strongly sub-feasible directions for solving problem (1.1), in which the initial point is arbitrary. And our algorithm possesses the following features:

- A new technique for constructing the submatrix $\bar{F}_k$ in the lower right corner of the coefficient matrix is presented, thus the coefficient matrix possesses good sparsity;
- A new working set technique is introduced, then only functions indexed by working sets are considered, which can reduce the scale of the subproblems;
- At each iteration, two SLEs with a same uniformly nonsingular coefficient matrix need to be solved, which further reduce the computation cost;
- Under mild conditions, the proposed algorithm has global and strong convergence.

The paper is organized as follows. The next section describes the algorithm. Section 3 discusses the convergence analysis. Section 4 contains numerical experiments. Finally, a conclusion is given in Section 5.