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A HYBRID VISCOSITY APPROXIMATION METHOD FOR A COMMON SOLUTION OF A GENERAL SYSTEM OF VARIATIONAL INEQUALITIES, AN EQUILIBRIUM PROBLEM, AND FIXED POINT PROBLEMS*

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Abstract

In this paper, we introduce a new iterative method based on the hybrid viscosity approximation method for finding a common element of the set of solutions of a general system of variational inequalities, an equilibrium problem, and the set of common fixed points of a countable family of nonexpansive mappings in a Hilbert space. We prove a strong convergence theorem of the proposed iterative scheme under some suitable conditions on the parameters. Furthermore, we apply our main result for W-mappings. Finally, we give two numerical results to show the consistency and accuracy of the scheme.

Mathematics subject classification: 47H10, 47J25, 47H09, 65J15, 46T20.
Key words: Equilibrium problem, Iterative method, Fixed point, Variational inequality.

1. Introduction

Let H be a real Hilbert space with inner product $\langle\cdot,\cdot\rangle$ and norm $\|\cdot\|$ and let C be a nonempty closed convex subset of H. A mapping T of C into itself is called nonexpansive if $\|Tx-Ty\|\le\|x-y\|$ for all $x,y\in C$. We use $Fix(T)$ to denote the set of fixed points T, i.e., $Fix(T)=\{x\in C: Tx=x\}$. Also, $f:C\to C$ is a contraction if $\|f(x)-f(y)\|\le\kappa\|x-y\|$ for all $x,y\in C$ and some constant $\kappa\in[0,1)$. In this case, f is said to be a κ-contraction.

Consider an equilibrium problem (EP) which is to find a point $x\in C$ satisfying the property:

$$\phi(x,y)\ge 0 \quad \text{for all } y\in C, \tag{1.1}$$

where $\phi: C\times C\to\mathbb{R}$ is a bifunction of C. We use $EP(\phi)$ to denote the set of solutions of EP (1.1), that is, $EP(\phi)=\{x\in C: (1.1) \text{ holds}\}$. The EP (1.1) includes, as special cases, numerous problems in physics, optimization and economics. Some authors (e.g., [12–14, 17–20, 22–24]) have proposed some useful methods for solving the EP (1.1). Set $\phi(x,y)=\langle Ax, y-x\rangle$ for all $x,y\in C$, where $A: C\to H$ is a nonlinear mapping. Then, $x^*\in EP(\phi)$ if and only if

$$\langle Ax^*, y-x^*\rangle\ge 0 \quad \text{for all } y\in C, \tag{1.2}$$

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that is, x^* is a solution of the variational inequality. The (1.2) is well known as the classical variational inequality. The set of solutions of (1.2) is denoted by $VI(A, C)$.

In 2008, Ceng et al. [5] considered the following problem of finding $(x^*, y^*) \in C \times C$ satisfying

$$\begin{cases} \langle \nu Ay^* + x^* - y^*, x - x^* \rangle \geq 0 & \text{for all } x \in C, \\ \langle \mu Bx^* + y^* - x^*, x - y^* \rangle \geq 0 & \text{for all } x \in C, \end{cases} \tag{1.3}$$

which is called a general system of variational inequalities, where $A, B : C \to H$ are two nonlinear mappings, $\nu > 0$ and $\mu > 0$ are two fixed constants. Precisely, they introduced the following iterative algorithm:

$$\begin{cases} x_1 = u \in C, \\ y_n = P_C(x_n - \mu Bx_n), \\ x_{n+1} = \alpha_n u + \beta_n x_n + \gamma_n SP_C(y_n - \lambda Ay_n), \end{cases}$$

where $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ are real sequences, S is a nonexpansive mapping on C, P_C is the metric projection of H onto C and obtained strong convergence theorem.

The implicit midpoint rules for solving fixed point problems of nonexpansive mappings are a powerful numerical method for solving ordinary differential equations. So, many authors have studied them; see [2, 7, 10, 16, 21] and the references therein. In 2015, Xu et al. [21] applied the viscosity technique to the implicit midpoint rule for nonexpansive mappings and proposed the following viscosity implicit midpoint rule:

$$x_{n+1} = \alpha_n f(x_n) + (1 - \alpha_n)T(\frac{x_n + x_{n+1}}{2}), \quad n \geq 0,$$

where $\{\alpha_n\}$ is a real sequence. They proved the sequence $\{x_n\}$ converges strongly to a fixed point of T which is the unique solution of a certain variational inequality.
Also, Ke and Ma [10] studied the following generalized viscosity implicit rules:

$$x_{n+1} = \alpha_n f(x_n) + (1 - \alpha_n)T(t_n x_n + (1 - t_n)x_{n+1}), \quad n \geq 0, \tag{1.4}$$

where $\{\alpha_n\}$ and $\{t_n\}$ are real sequences. They showed the sequence $\{x_n\}$ converges strongly to a fixed point of T which is the unique solution of a certain variational inequality.

Recently, Cai et al. [4] introduced the following modified viscosity implicit rules

$$\begin{cases} x_1 \in C, \\ u_n = t_n x_n + (1 - t_n)y_n, \\ z_n = P_C(I - \mu B)u_n, \\ y_n = P_C(I - \lambda A)z_n, \\ x_{n+1} = P_C(\alpha_n f(x_n) + \beta_n x_n + ((1 - \beta_n)I - \alpha_n \rho F)Ty_n), \quad n \geq 1, \end{cases}$$

where F is a Lipschitzian and strongly monotone map, $\{\alpha_n\}$, $\{\beta_n\}$ and $\{t_n\}$ are real sequences, P_C is the metric projection of H onto C. Under some suitable assumptions imposed on the parameters, they obtained some strong convergence theorems.

In this paper, motivated by the above results, we propose a new composite iterative scheme for finding a common element of the set of solutions of a general system of variational inequalities, an equilibrium problem and the set of common fixed points of a countable family of nonexpansive mappings in Hilbert spaces. Then, we prove a strong convergence theorem and apply our main result for W-mappings. Finally, we give two numerical examples for supporting our main result.