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MONOLITHIC AND PARTITIONED FINITE ELEMENT SCHEMES FOR FSI BASED ON AN ALE DIVERGENCE-FREE HDG FLUID SOLVER AND A TDNNS STRUCTURAL SOLVER

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Abstract. We present novel (high-order) finite element schemes for the fluid-structure interaction (FSI) problem based on an arbitrary Lagrangian-Eulerian divergence-free hybridizable discontinuous Gakerkin (ALE divergence-free HDG) incompressible flow solver, a Tangential-Displacement-Normal-Normal-Stress (TDNNS) nonlinear elasticity solver, and a generalized Robin interface condition treatment. Temporal discretization is performed using the high-order backward difference formulas (BDFs). Both monolithic and strongly coupled partitioned fully discrete schemes are obtained. Numerical convergence studies are performed for the flow and elasticity solvers, and the coupled FSI solver, which verify the high-order space-time convergence of the proposed schemes. Numerical results on classical two dimensional benchmark problems also showed good performance of our proposed methods.

Key words. Divergence-free HDG, ALE, FSI, TDNNS, generalized Robin condition, partitioned scheme.

1. Introduction

Fluid-structure interaction (FSI) describes a multi-physics phenomenon that involves the highly non-linear coupling between a deformable or moving structure and a surrounding or internal fluid. There has been intensive interest in numerically solving FSI problems due to its wide applications in biomedical, engineering and architecture fields [18, 32, 48].

Based on different temporal discretization strategies, the numerical procedure to solve FSI problems can be broadly classified into two approaches, see, e.g., [76]: the *monolithic approach* and the *partitioned approach*. The monolithic approach [5, 40, 49, 65, 73, 95] solves for the fluid flow and structural dynamics simultaneously by a unified algorithm. Since the interfacial conditions are *automatically* satisfied in the solution procedure, monolithic schemes allow for more rigorous analysis of discretization and solution techniques, and are usually more robust than partitioned schemes. On the other hand, the partitioned approach [30, 34, 66] gains computational efficiency over the monolithic approach by solving the fluid and structure sub-problems separately in a sequential manner, usually with the help of a proper *explicit* coupling condition on the fluid-structure interface to separate the fluid and structure solvers. But the design of efficient partitioned schemes that produce stable and accurate results remains a challenge, especially when the fluid and structure have comparable densities, as it happens in hemodynamic applications, due to numerical instabilities known as *the added mass effect* [17]. The design and analysis of partitioned numerical methods that address the added mass effect remains an active research area in the past decade, see, e.g., [1, 3, 4, 10, 14, 39] and references cited therein.

The finite element method is one of the most popular choices for the numerical simulation of FSI problems [6, 11, 12, 86]. Of particular relevance to the current

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contribution is the class of discontinuous Galerkin (DG) finite element schemes, which has gained increased interest in the computational fluid dynamics community [21, 27, 29] due to their distinctive features, such as the ability to achieve high-order accuracy on complex geometries using unstructured meshes and meshes with general polygonal/polyhedral elements, the flexibility in performing h- and p-adaptivity, the local conservation property, and the upwinding stabilization mechanism for stabilizing dominant convection effects.

One of the major difficulties in nonlinear FSI problems stems from the movement of the fluid domain, which makes these problems computationally very challenging, where the major bottleneck is a robust fluid flow solver on deforming domains. Various techniques have been introduced for fluid problems with moving boundaries and interfaces, which include the interface-tracking approaches, e.g., the arbitrary Lagrangian-Eulerian (ALE) method [31, 50] and the space-time method [59, 85, 89] where the computational mesh tracks and fits with the moving interfaces, and the interface-capturing approaches, e.g., the immersed boundary method [60, 72], the immersed finite element method [57, 94], the fictitious domain method [46], and the extended/generalized finite element method [19, 41], where the computational mesh is static and does not fit to the moving interfaces. The current work focuses on the ALE approach for the fluid solver; see, e.g., [38, 58, 70] for ALE-DG schemes for compressible flow problems, and [36, 90] for incompressible flow problems.

There have been a few ALE-DG fluid solver based schemes for FSI problems, see, e.g., [37, 42, 71] where the nonlinear structure equations were discretized using the standard conforming Galerkin (CG) method, and [54, 90] where the structure equations were also discretized using DG methodologies. We also cite the related work [2] on space-time DG FSI solvers. One major criticism of DG schemes for problems involve linear system solvers is their associated high computational cost when compared to standard CG schemes, mainly due to a larger number of (element-based) degrees of freedom (DOFs) and the element-element DOFs couplings in the resulting linear system problem. The hybridizable discontinuous Galerkin (HDG) methods [20, 25, 64] were introduced to try to address this criticism. Basically, HDG schemes introduce facet-based DOFs on the mesh skeleton so that element-element DOFs couplings in the standard DG schemes are replaced by facet-element couplings, which result in a reduced globally coupled linear system involves facet-based DOFs only after a *static condensation* procedure that locally eliminates the (local) element-based DOFs. Hence the computational cost of HDG schemes are usually much lower than that of the DG schemes, especially for high-order approximations [53, 93]. Besides being computationally cheaper, the HDG methods also produce more accurate approximations than DG methods for certain problems due to their superconvergence property [22–24, 74].

The first HDG scheme for FSI problems was introduced in [81], where the authors combined the HDG incompressible flow and elasticity solvers [64] with a monolithic ALE formulation. The method was further improved in [82] with a reduced computational cost that uses a more efficient elasticity HDG solver and a linear finite element approximate for the ALE map. More recently, an ALE partitioned scheme [55] based on an HDG formulation for the compressible fluid and a CG formulation for the structure has been proposed for FSI problems with a weakly compressible fluid.

For incompressible flow problems, numerical discretizations that yield point-wise divergence-free velocity approximations have attracted increased attention [51], due