Commun. Comput. Phys.
doi: 10.4208/cicp.OA-2022-0171

Vol. **33**, No. 3, pp. 884-911
March 2023

Numerical Reconstruction of Locally Rough Surfaces with a Newton Iterative Algorithm

Meng Liu[1] and Jiaqing Yang[1,*]

[1] *School of Mathematics and Statistics, Xi'an Jiaotong University, Xi'an 710049, P.R. China.*

Received 23 June 2022; Accepted (in revised version) 16 January 2023

Abstract. In this paper, we propose a Newton iterative algorithm to numerically reconstruct a locally rough surface with Dirichlet and impedance boundary conditions by near-field measurements of acoustic waves. The algorithm relies on the Fréchet differentiability analysis of the locally rough surface scattering problem, which is established by reducing the original model into an equivalent boundary value problem with compactly supported boundary data. With a slight modification, the algorithm can be also extended to reconstruct the local perturbation of a non-local rough surface. Finally, numerical results are presented to illustrate the effectiveness of the inversion algorithm with the multi-frequency data.

AMS subject classifications: 35R30, 35J05, 35P25, 65N21

Key words: Newton iterative algorithm, Fréchet derivative, inverse scattering, locally rough surface, Dirichlet condition, impedance condition, multi-frequency data.

1 Introduction

The inverse scattering problems are concerned with determining the nature of an unknown scatterer such as the shape, the position or material properties from the scattered field measurements. Such kind of problems has a wide range of important applications, for example, remote sensing, nondestructive evaluation and geophysics, e.g., [3, 5, 14, 15, 24, 36]. To obtain an approximate solution of the inverse problem, one popular way is to develop iterative inversion algorithms whose foundation is to analyze the Fréchet differentiability of the boundary or physical parameters to the far-field or near-field operator. Lots of works have been devoted to this topic especially for the shape

*Corresponding author. *Email addresses:* liumeng13@stu.xjtu.edu.cn (M. Liu), jiaq.yang@mail.xjtu.edu.cn (J. Yang)

identification problems associated with inverse time-harmonic acoustic, elastic and electromagnetic wave scattering. We refer the reader to the works on the acoustic scattering by Hettlich [19, 20] and Kirsch [22] via the variational method, Hohage [21] via an implicit function theorem, [29,31,33] via the boundary integral equation method, and Kress & Päiärinta [23] via a factorization of the difference of the scattered solutions for various boundary conditions. Partial results were also extended to the elastic and electromagnetic scattering by Charalambopoulos [11], Le Louër [25] and Potthast [32], Haddar & Kress [18]. We remark that all works mentioned above are focused on recovering the shape of a bounded obstacle.

In this paper, we investigate the inverse acoustic scattering of time-harmonic point sources by a locally rough surface with Dirichlet and impedance boundary conditions. It is known that the well-posedness of the forward problem has been extensively studied in the literature by the boundary integral equation method, variational approaches or the coupling of the above two methods; see e.g., [1, 3, 9, 17, 35, 39]. We also refer the reader to the acoustic scattering by global rough surfaces where the classical Sommerfeld radiation condition has to be replaced by a weak upward propagating radiation condition proposed by Chandler-Wilde & Zhang *et al.* [7,10,37,38]. Compared to the forward problem, the inverse problem we are interested in is to determine a locally rough surface by taking the near-field measurements. The objective is to propose a Newton iterative algorithm of reconstructing the shape and location of the locally rough surface. Although many publications can be found for iterative inversion algorithms with a Fréchet differentiability analysis, most of those are confined to the inverse scattering by bounded obstacles. It is more challenging to investigate related inversion algorithms for the inverse scattering problem associated with an infinite rough surface. So far, very little result is available in the literature for this case. It was shown in [3] an iterative inversion algorithm was proposed by Bao & Lin for the Dirichlet boundary value problem in the sense of a domain derivative via the variational method, where a key tool is to use the reflection principle for the Helmholtz equation with respect to a planar surface. However, it should be noticed that the reflection principle is not valid for the impedance boundary value problem as well as nonlocal rough surface scattering problems. To the best of our knowledge, no results are available in the literature for these cases. The main mathematical contribution of this paper is that we proposed a novel technique to establish the differentiability analysis for locally rough surface scattering problems associated with the Dirichlet and impedance boundary conditions in a consistent manner. Moreover, with a very slight modification, our method can be extended to establish the Fréchet differentiability on the local perturbation of a nonlocal rough surface with various boundary conditions. To this end, inspired by the work of Ding *et al.* [16] on the locally rough surface problem, we first reduce the original model by introducing a special rough surface into an equivalent boundary value problem with compactly supported boundary data, for which the well-posedness can be easily proved via the boundary integral equation formulation defined on a bounded arc. Then the Fréchet differentiability can be established for both Dirichlet and impedance boundary value problems based on repeated uses of Green's theorem