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The Acoustic Scattering of a Layered Elastic Shell Medium

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Dedicated to the memory of Professor Zhongci Shi

Abstract. This paper investigates the scattering of a three-dimensional elastic shell scatterer embedded in a layered unbounded structure. The background structure comprises a two-layer lossy media separated by an unbounded rough surface. The shell body is filled with an elastic material, the interior of which is vacuum. Given an incident acoustic point source, our aim is to determine the acoustic and elastic wave fields in the space. This problem is known as a fluid-solid interaction problem (FSIP). In this work, the uniqueness of the FSIP solution is proved by using the integral equation method. Based on the decay properties of Green's function, the equivalence of boundary integral equation systems and the FSIP is established. Since the system of integral equations containing several integral operators on infinite intervals, we introduce the index theorem of integral operators and analyze the integral operators on infinite intervals to prove the system of integral equations is Fredholm. We then obtain the uniqueness of the system of integral equations and the corresponding existence and uniqueness result of the FSIP.

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Key words: Fluid-solid interaction problem, Helmholtz equation, Navier equation, integral operator, index theorem, existence and uniqueness.

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1 Introduction

As the shell structure is often used in underwater targets, the acoustic scattering problem of underwater elastic shells is the foundation of target reconstruction in the field of underwater acoustics. A large number of studies dealing with various aspects of the transient response of submerged and/or fluid-filled elastic shell structures with simple geometrical configuration (closed-form analytic solution for spherical or cylindrical shells) have been reported in the literature. The reader is referred to [7, 8, 14] and the references therein.

This paper considers a time-harmonic acoustic wave incident onto an interface of the two-layered medium from the above. The medium above the surface is supposed to be filled with some homogeneous fluid with a constant mass density, such as air, where the region below is occupied by another homogeneous fluid (such as water) containing a submerged shell with a general shape. An acoustic wave incoming through the interface from the air into the water is incident on the elastic shell, which generates an elastic wave inside the shell body. The process further yields acoustic waves scattered in the interior of the shell, which is the fluid-solid interaction on the shell between acoustic and elastic waves.

For the scattering of fluid-solid interactions, often a linear elastic bounded obstacle surrounded by an inviscid compressible fluid is considered. A Kirchhoff-type formula for transient elastic waves was originally sketched by Love in [10]. Helmholtz and Kirchhoff type integral formulas were systematically derived for elastic waves in isotropic and anisotropic solids by Pao and Varatharajulu [13]. In the region where the wavelength is of the same order of magnitude as the period and much greater than the depth of the grating, theoretical and numerical results concerning diffraction of ultrasonic waves on periodic liquid–elastic solid interfaces were presented [6]. Various systems of boundary integral equations over the interface between the fluid and the solid were derived and analysed by Luke and Martin [11]. Hsiao, Kleinman and Roach presented weak formulations of the coupled problem using the field equations in the solid and boundary integral equations in the fluid. They applied some appropriate function spaces and obtained that the weak formulation in terms of a sesquilinear form which is not strongly coercive but for which the Gärding's inequality holds [9]. Hsiao and Yin considered a regularization formulation of the hypersingular operator for the two-dimensional fluid-solid interaction problem [16].

There are also recent results on the related acoustic or elastic scattering of an unbounded rough surfaces. When an unbounded interface is considered, additional difficulties arise. Arens applied the integral equation method to prove the existence of a solution in elastic wave scattering by unbounded rough surfaces [1, 2]. More recently, homogeneous obstacle acoustic composite scattering problems have been