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Low Rank and Total Variation Based Two-Phase Method for Image Deblurring with Salt-and-Pepper Impulse Noise

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Abstract. Although there are many effective methods for removing impulse noise in image restoration, there is still much room for improvement. In this paper, we propose a new two-phase method for solving such a problem, which combines the nuclear norm and the total variation regularization with box constraint. The popular alternating direction method of multipliers and the proximal alternating direction method of multipliers are employed to solve this problem. Compared with other algorithms, the obtained algorithm has an explicit solution at each step. Numerical experiments demonstrate that the proposed method performs better than the state-of-the-art methods in terms of both subjective and objective evaluations.

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1. Introduction

Impulse noise removal is a challenging problem in image restoration. In general, the image restoration problem, which is subject to blurring and impulse noise, can be expressed as

$$f = N_{imp}(y), \quad y = Kx, \tag{1.1}$$

where N_{imp} denotes impulse noise, K is a linear blurring operator, $f \in R^{m\times n}$ is the observed image, and $x \in R^{m\times n}$ is the unknown true image. Two types of impulse noise are widely studied: salt-and-pepper (SP) impulse noise and random-valued (RV) impulse noise. Let the dynamic range of y belong to $[y_{\min}, y_{\max}]$, i.e., $y_{\min} \le y_{ij} \le y_{\max}$, for all $1 \le i \le m$, $1 \le j \le n$.

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Salt-and-pepper impulse noise. Observed image f satisfies

$$f_{ij} = \begin{cases} y_{\min}, & \text{with probability } s/2, \\ y_{\max}, & \text{with probability } s/2, \\ y_{ij}, & \text{with probability } 1-s, \end{cases} \tag{1.2}$$

where s denotes the level of the salt-and-pepper noise.

Random-valued impulse noise. Observed image f satisfies

$$f_{ij} = \begin{cases} d_{ij}, & \text{with probability } r, \\ y_{ij}, & \text{with probability } 1-r, \end{cases} \tag{1.3}$$

where d_{ij} are uniformly distributed random numbers in $[y_{\min}, y_{\max}]$ and r denotes the level of the random valued noise.

The most popular model for image deblurring with impulse noise is the so-called L1-TV, which is defined by

$$\min_x \ \|Kx - f\|_1 + \lambda\varphi(Lx), \tag{1.4}$$

where $\lambda > 0$ is the regularization parameter, $L : R^{m\times n} \to R^{m\times 2n}$ is the first-order difference matrix, and $\varphi : R^{m\times 2n} \to R$ is a convex function. If $\varphi(\cdot) = \|\cdot\|_2$ or $\varphi(\cdot) = \|\cdot\|_1$, $\varphi(Lx)$ denotes the isotropic total variation (ITV) and the anisotropic total variation (ATV), respectively. The first term in (1.4) is usually called data fidelity term, and the second term is called regularization term. Compared with the classical L2-data fidelity term, the L1-data fidelity term is robust for removing outliers. The L1-TV model (1.4) is difficult to solve because of the nondifferentiable of both the L1-norm data fidelity term and the TV term. In the last two decades, many efficient iterative algorithms have been proposed to solve (1.4). These include the primal-dual interior point algorithm [21], alternating minimization algorithm [23], alternating direction method of multiplies [12, 34], and the primal-dual Chambolle-Pock algorithm [6].

Although the L1-TV model (1.4) is effective in removing impulse noise, it does not take into account whether a pixel is contaminated by noise or not. The performance of L1-TV is usually unsatisfactory when the noise level is high, as demonstrated in studies such as [13, 36]. To address this issue, two-phase methods have gained popularity. In the first phase, techniques such as the adaptive median (AM) filter or adaptive center-weighted median (ACWM) filter are used to identify image pixels affected by salt-and-pepper impulse noise or random-valued impulse noise. In the second phase, filter methods or detail-preserving regularization methods based on the identified noise-free pixels are utilized to recover the image. Chan *et al.* [9–11] first proposed a two-phase method for removing random-valued impulse noise and salt-and-pepper impulse noise, respectively. They considered solving a variational minimization problem in the second phase, see also [4, 7, 17]. On the other hand, Chen and Yang [16] introduced a two-stage method for removing impulse noise, which used an iterative