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Modified Differential Transform Method for Solving Black-Scholes Pricing Model of European Option Valuation Paying Continuous Dividends

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Abstract. Option pricing is a major problem in quantitative finance. The Black-Scholes model proves to be an effective model for the pricing of options. In this paper a computational method known as the modified differential transform method has been employed to obtain the series solution of Black-Scholes equation with boundary conditions for European call and put options paying continuous dividends. The proposed method does not need discretization to find out the solution and thus the computational work is reduced considerably. The results are plotted graphically to establish the accuracy and efficacy of the proposed method.

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1 Introduction

The financial agreements made between buyers and sellers in the financial markets is known as option pricing. It accounts for numerous purposes for example to hedge assets and portfolios to minimize or to control the risk due to variability in stock prices. There are varied mathematical models available for pricing different types of options. The problem for pricing options was mathematically modeled first in 1973 by Fisher Black and

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Myron Scholes [1] on European or American call (right to buy) and put (right to sell) options. European options are traded only on a specified future date before expiry while American options are exercised or traded at any time up to the expiry.

The Black-Scholes model is mainly based on the concept of constructing a risk-less portfolio taking into account the bonds, option and the underlying stock. Moreover, it also takes into consideration the concept of hedging and eliminating risk of option pricing for purchasing and selling of underlying assets. The Black-Scholes option pricing model is based on the assumption that the underlying asset does not pay any dividends during the life time of the option. Merton [2] extended the Black-Scholes option pricing model to underlying assets that pay a continuous dividend yield during the life time of the option and derived the modified Black-Scholes equation and the modified Black-Scholes formulae for both European call and put options. The Black-Scholes model for the valuation of European options paying continuous dividend yield is given by the partial differential equation:

$$\frac{\partial V}{\partial t}+\frac{\sigma^2 S^2}{2}\frac{\partial^2 V}{\partial S^2}+(r-D)S\frac{\partial V}{\partial s}-rV=0, \tag{1.1}$$

where $V(S,t)$ is the option price or option premium at asset price S and at time t, $S(t)$ is the asset price at time t, σ represents the volatility, r is the risk free interest rate and D is the dividend yield.

Let us denote the value of European call and put options by $V_c(S,t)$ and $V_p(S,t)$ respectively. Then the pay off functions for European call and put options are given by:

$$V_c(S,t)=\max(S-K,0) \quad \text{and} \quad V_p(S,t)=\max(K-S,0), \tag{1.2}$$

where K denotes the strike price. It is well known that Eq. (1.1) has a closed form solution depending on the fundamental solution of heat equation. Hence, it becomes necessary to transform the Back-Scholes equation to a heat equation after certain change of variables. After finding the closed form solution of the heat equation, it can be transformed back to find the solution of the Black-Scholes partial differential equation [3].

Since the classical Black-Scholes equation was established under some strict assumptions, a wide class of analytical and numerical methods has been proposed from time to time to weaken these assumptions such as Laplace method [4, 5], Fourier transform method [6], iteration method [7], homotopy perturbation method [8–10], variational iteration method [11], new iterative method [12], homotopy analysis method [13], Adomian decomposition method [14] and differential transform method [15].

The main aim of this paper is to extend the application of modified differential transform method to obtain the approximate analytical solution of Black-Scholes European call and put options paying continuous dividends. The differential transform method was first applied by Zhou in 1986 in the field of engineering to solve linear and nonlinear equations in electrical circuit analysis [16]. Several authors have further modified the differential transform method and applied it to certain differential equations both