Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **16**, No. 1, pp. 181-207

DOI: 10.4208/aamm.OA-2022-0085
February 2024

A Vertex-Centered Arbitrary Lagrangian-Eulerian Finite Volume Method with Sub-Cells for Two-Dimensional Compressible Flow

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Received 23 March 2022; Accepted (in revised version) 14 October 2022

Abstract. In this paper, we present a new vertex-centered arbitrary Lagrangian-Eulerian (ALE) finite volume scheme for two-dimensional compressible flow. In our scheme, the momentum equation is discretized on the vertex control volume, while the mass equation and the energy equation are discretized on the sub-cells which are included in the vertex control volume. We attain the average of the fluid velocity on the vertex control volume directly by solving the conservation equations. Then we can obtain the fluid velocity at vertex with the reconstructed polynomial of the velocity. This fluid velocity is chosen as the mesh velocity, which makes the mesh move in a Lagrangian manner. Two WENO (Weighted Essentially Non-Oscillatory) reconstructions for the density (the total energy) and the velocity are used to make our scheme achieve the anticipated accuracy. Compared with the general vertex-centered schemes, our scheme with the new approach for the space discretization can simulate some multi-material flows which do not involve large deformations. In addition, our scheme has good robustness, and some numerical examples are presented to demonstrate the anticipated accuracy and the good properties of our scheme.

AMS subject classifications: 65M60

Key words: Vertex-centered, arbitrary Lagrangian-Eulerian, sub-cells, multi-material flows.

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1 Introduction

Simulations of the compressible multi-material fluid flows can be classified into two computational frameworks, each with its own advantages and disadvantages. The first one is Lagrangian [1–4] in which the mesh is embedded in fluid and moves with it; the second, known as Eulerian [5,6] treats the mesh as a fixed reference frame through which the fluid moves. In order to combine the advantages of the Lagrangian method and the Eulerian method, Hirt et al. [7] proposed an Arbitrary Lagrangian-Eulerian (ALE) method in which the grid points may move in some arbitrarily specified ways. The ALE method is a powerful tool to solve fluid dynamic problems with large deformation. It was primarily developed from the finite difference method for fluid flows and later extended in the context of the finite element method and the finite volume method for both fluid and structure dynamics. Since the computational mesh of the ALE method can move with arbitrary velocity which is independent of the fluid velocity, the best features from the traditional Lagrangian method and the Eulerian method can be combined and embodied in the ALE method. Basically, there are two categories of the ALE methods which are the so-called indirect method [8–14] and the direct one [15–19]. In the indirect ALE method, the computational mesh firstly moves in a Lagrangian manner. When the mesh becomes too distorted, a remeshing step is carried out to generate a new grid with better quality. Then the numerical solutions on the old mesh are projected to the new one, which is called the remapping step. This method has been widely used for solving the multi-phase flow problems and the multi-material flow problems. For instance, Maire et al. [12] developed a cell-centered multi-material indirect ALE scheme for solving the compressible gas dynamics equations on two-dimensional unstructured grid, and the multi-material simulations utilized either the concentration equations for miscible fluids or the Volume Of Fluid (VOF) capability with interface reconstruction for immiscible fluids; Barlow and Maire et al. [14] reviewed the recent developments in the indirect ALE methods for modeling the high speed compressible multi-material flows on complex geometry with general polygonal meshes. In the direct ALE method, the mesh movement is taken into consideration directly in the computation of the numerical flux. Therefore, this method has no remapping step, and it is easier to be used for constructing high-order accuracy schemes than other methods with the remapping step. For instance, Zhao et al. [19] presented a high-order direct ALE Discontinuous Galerkin (DG) method for compressible single-material flow on the adaptive moving unstructured meshes; Boscheri et al. [17] proposed a high-order accurate ADER schemes in the direct ALE context.

There are two commonly used approaches for discretizing the gas dynamic equations. One is the staggered-grid hydrodynamics (SGH) [20–23] in which the velocity is defined at nodes while the other variables (the density, the pressure and the specific internal energy) are located inside the cells. The other is the cell-centered hydrodynamics (CCH) [1,2,4,17,24–34] in which all the primary variables are defined in the cells. The CCH method offers some advantages over the SGH one. For example, the SGH schemes use different control volumes for the primary variables, and it is difficult to